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Dynamic behavior analysis of composite rotor

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*Those who gave me life, pride, wisdom and patience, those who show me the responsibility, those who are the source of my courage, my inspiration, and my happiness: "**My dear parents**"*

" I love you until the last day of my life, and

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GENERAL INTRODUCTION

Present day, rotors have predominantly metallic shafts. Use of fiber-reinforced composites has been attempted in a few specific applications. The rotating composite material shafts are used as structural elements in many application areas involving the rotating machinery systems. This is likely to contribute to the high strength to weight ratio, lower vibration level, and longer service life of composite materials.

A significant weight saving can be achieved by the use of composite materials. Also by appropriate design of the composite lay-up configuration, orientation, and number of plies, the improve performance of the shaft system can be obtained. Furthermore, the use of composite would permit the use of longer shafts in the supercritical range than what it is not possible with conventional metallic shafts. In the last few years, there existed numerous researchers who their study was about predicting critical speeds and natural frequencies of composite shaft.

In the early developments, composite shafts were designed to operate in the subcritical range. Therefore, initial studies were directed toward to the design requirements and in overcoming the problems in practical application. Subsequently, in order to derive greater advantage in terms of reduction of weight, the possibility of super-critical operations of composite shafts was explored.

The design aspects for composite shafts vary from one application to another. Initially, the materials used for the drive shafts were glass/epoxy and boron/epoxy. However, as developments in composite technology continued, carbon fibers became more readily available.

In many current applications, they replaced boron fibers, which it had proved that it was too costly and difficult to process. In automotive drive shafts, carbon fibers were found to be particularly suitable. Hybridization with glass/epoxy was tried, and it proved cost effective too.

One of the problems associated with design of composite drive shafts has been the accurate determination of the flexural critical speeds. As the drive shafts are quite long, their critical speeds are lower and may occur near the operating speed. In order to analyze the problems related to the lateral bending of composite shafts, equivalent modulus theory is commonly used.

The theory is based on Kirchoff's hypothesis for thin laminated beams. The equivalent moduli are found using classical laminate theory.

This work is organized in three chapters, starting with the state of the art of the first chapter to the last conclusion of the third one, chapter one is considered as an introductory of the subject of this work, it collects briefly everything related to rotor dynamics and composite materials.

Chapter two presents the theoretical elements used to obtain the equations of motion of a rotational system, furthermore, it improves the energy expressions into generalized Timoshenko form, also, it focuses on the development of the equations of motion using the finite element method in order to determine the elementary matrices of each element of the rotor system.

The third chapter focuses on the presentation of the MATLAB program by representing the results of critical speeds and vibration amplitudes of two different composite materials, after sitting all their properties.

CHAPTER I
Bibliographic studies

I.1 State of the art

The knowledge of the dynamic behavior of composite rotors is of great importance in power production engineering and in adjacent fields. The subject of high-performance composite materials is involved in development modern design and manufacturing methods for industrial components.

This development requires the implementation of the tools necessary for modelling the mechanical behavior of composite materials. This introductory chapter discusses the foundation by introducing the basic rotor and structural dynamic terminologies, concepts and characteristics, also, it presents a global vision of the state of the art in the field of stability of rotating machines.

I.2 Problematic and objective

In the field of rotor dynamic, there are several problems and obstacles can be found throughout the studies of vibrations which lead to the damage of the rotors. Our studies are specialized in the field of dynamic analysis of composite rotors, in order to solve those problems and to find solutions for the latter. Moreover, and as it’s known that the composite materials are characterized by very high mechanical properties which make them able to with stand the applied solicitation more than a normal material.

I.3 History

Studies on composite shafts started in 1970’s. The most important development of composite shafts has taken place in aerospace (helicopter) industry [1] [2] and automotive applications [3] [4]. Other applications include the use of composite shafts as quill shaft by Spencer [5], an aircraft power take off shaft by Garguilo [6], generator shaft by Raghava and Hammond [7], shaft for a cooling tower by Berg [8], a papermill by Cox [9] and naval propulsion systems by Wilhelmi et al [10]. The two U.S. patents by Worgan and Smith [11] and Yates and Rezin [12] indicate that the preliminary hurdles to a composite driveshaft design were overcome. Fromknecht [13] highlighted the possible benefits accruing from the use of composite shaft in mechanical power transmission.

From 1970’s till today, the studies over composite rotors have passed by so many stages :

- Zinberg and Symonds [14] investigated the critical speeds of rotating anisotropic cylindrical shafts based on an equivalent modulus beam theory (EMBT), and dos Reis et al. [15] evaluated the shaft of Zinberg and Symonds by the finite element method.
- Bauchau [16]: specialized in studies of high-speed graphite/epoxy shafts and the design of tapered composite shafts. The study of Bauchau deal with a numerical procedure using a generalized criterion of optimality. The shaft was modeled using a beam formulation including shear deformation and rotary inertia. The optimization was done under constant volume constraint. Different configurations are studied for various number of plies and orientation angle. In the optimal configuration the natural frequency increased by about 21 to 44%, and the bending stress decreased by about 48 to 59%.
- Spencer and Mcgee [17]: worked on the design of subcritical composite drive shafts in order to overcome its hurdles. Their report contains an analysis of the present steel shaft design

  and the two design approaches to a composite drive shaft. The first is a two-piece drive shaft; the second is a combined single drive shaft eliminating several other parts. The design section includes the design of a joint to interface between the steel couplings and the composite shaft.
- Zorzi and Gioradani [18]: by the experiments on an aluminum shaft and on a composite shaft, they made an achievement on matching the experimental and theoretical results.
- Lim and Darlow [19]: the reduction in weight of the rotor system led them to look forward if there is a possibility of supercritical operations. The optimal design of composite drive shafting is developed with the goal of minimizing, system weight. The study is illustrated with an application to a composite tail rotor drive shaft for advanced helicopters. It is also applicable to the design of composite synchronization drive shah for helicopters and other composite shah for aircraft, spacecraft and automobiles. The use and effectiveness of the optimal design procedure are demonstrated with an illustrative example.
- Reis and all [15]: used the finite element method to evaluate the critical speeds of composite shafts. A numerical procedure to evaluate the rotor dynamic performance of thin-walled filamentary wound laminated composite circular cylindrical shafts of any layup is presented. Numerical results, for the critical speeds and the unbalance response of a sample composite shaft, are obtained and compared with predictions based on classical methods of analysis and experimental results found in the literature.

- Hoffman [20]: proved that carbon fiber is necessary to have a balance between length, diameter and natural frequency. Another problem of paramount importance in composite shaft design has been that of optimization of the material and geometric parameters. The optimization objectives are somewhat different in aerospace applications as compared to automotive driveshaft design. In automotive applications, cost is one of the major driving factors. Thus, detailed cost-sensitivity analyses are performed in order to get a cost-optimal design. The solution lies in using hybrid composite shafts, as shown by Hoffmann.
  This provides the engineer with two important design variables to control, viz, the fiber winding angle and the mixing ratio of carbon and glass fibers. The additional variable of carbon-glass ratio greatly increases the range of designs alternatifs.
- Hetherington and all [21]: also worked on the reduction in weight but of the tail drive rotor because they studied on composite helicopter. An experimental program is underway to investigate the dynamic behavior of supercritical composite drive shafts for helicopter applications. Design optimization results have shown that the system of least weight is achieved by the use of composite materials with a shaft that operates at a supercritical speed. Uncertainties in the ability to manufacture, balance, and safely operate supercritical composite shafts motivates the experimental program to examine their dynamic performance. Results are presented far experiments with both aluminum and optimized graphite/epoxy shafts.
- Singh and Gupta [22] : The shell modes involving cross sectional deformation of nonrotating shafts have been experimentally analyzed by modal testing by Singh and Gupta Tests showed the existence of coupling of higher flexural modes with shell modes. The mounting
  of a disc on the shaft resulted in suppression of some shell modes, reduction in flexural natural frequencies and increase in damping ratios of all modes.

I.4 Rotors modeling

In 1919, « Jeffcott » studied the basics about the dynamics of rotating machines, his work led to the development of essential studies such as Campbell diagram. A simple rotor dynamic model is known as « Jeffcott rotor ». Any element rotating around a fixed axis is a rotor [23].

Its basic elements are: the shafts, the discs and the bearings, such that, the discs are defined as active parts mounted on the shafts, and the bearings are rotary connections in which the rotor is held. The complete rotor modeling is depending on the specific modeling of these elements [24].

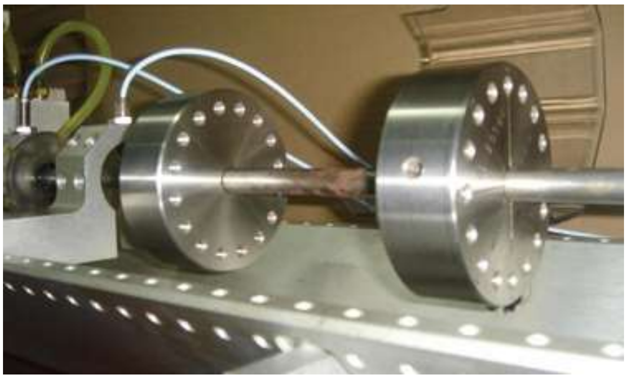

Figure I. 1 : A close view of a rotor consisting of two discs mounted on a flexible shaft [25].

1) The shaft

The shaft is a beam with a circular form, where the disc is based on.

There are some reasons that make the choice of its form necessary, like « form guide and assembly surfaces, resistance, cost price » [24].

In addition, we find two principals types of shafts « according to its form and function »:

- Transmission shafts.
- Machine shafts.

2) The disk

The disk is defined as a solid and axially slim wheel, on which mechanical work is performed or from which work is extracted [26] .

3) The bearing

The bearing is considered as a support for the rotor, it is classified in two categories: fluid-film bearing and rolling-element bearing, it provides dynamic constraint in both directions « transverse and axial » [26].

4) The unbalance

Even that the masse of unbalance is negligible compared to the masse of the rotor [27] there is always a residual unbalance related to rotors no matter how they are balanced [28] , and it is all because of the results of manufacturing tolerances, operational wear and tear, thermal

distortions [25], the unbalance is created, therefore, it has to be considered into the modeling [24].

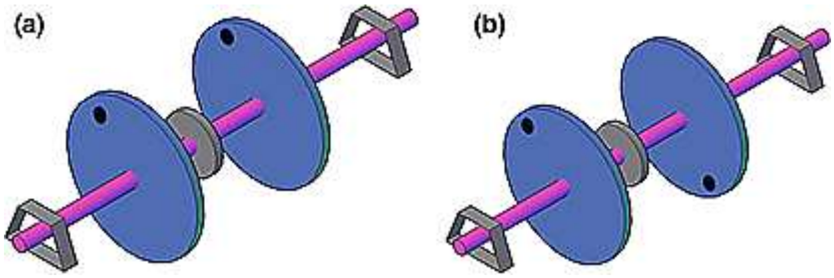

Figure I. 2: (a) Static unbalance (b) Dynamic unbalance [29].

I.5 Characterizations of the elements of rotor

The influence on the dynamic behavior of the rotor appears from the characteristics of the rotor elements (shaft, disc, bearing), their changes also influence on the dynamic equations of system. There are two parameters, which can define these characteristics [30] :

1) **Geometric parameters**
   - Uniformity of the shaft (eg: diameter variation)
   - Disk thickness
   - Presence of discontinuity (eg: cracked rotor)
   - The nature and the types of bearings

2) **Mechanical parameters**
   - Disk stiffness or flexibility.
   - Disk flexibility effect.
   - The nature of the shaft material.
   - Isotropy and anisotropy.
   - The bearing mouvements.

I.6 Different types of rotors

In general, there are two special classes of rotos :

1) Rigid rotor

A rigid rotor is defined as the one :

- Which operates below its first bending critical speed [26].
- That has one frequency prior to the operating or rated speed of a machine [29]

Throughout all the operating speed range, these rotors may be balanced on any two arbitrarily selected correction planes, also, they can be brought into and will remain in state of balance [26] [31].

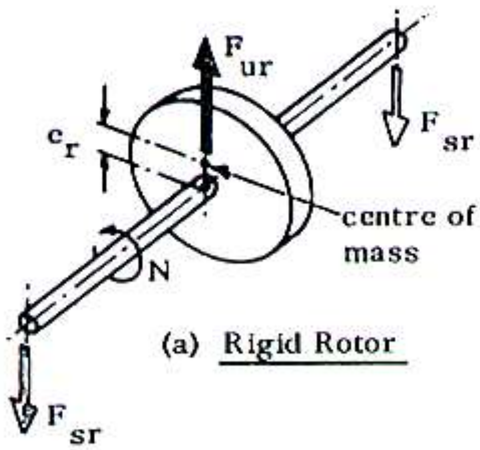

Figure I. 3: Rigid rotor [32].

2) Flexible rotor

A flexible rotor is known as the one :

- That cannot be balanced in a low-speed balancing machine [31] .
- That has more than one frequency to balance before reaching to the operating speed [29].

For dynamic effects and in order to influence rotor deformations, these rotors operates close to or beyond its first bending critical speed [26], moreover, they require one or more high speed trim plane correction [31].

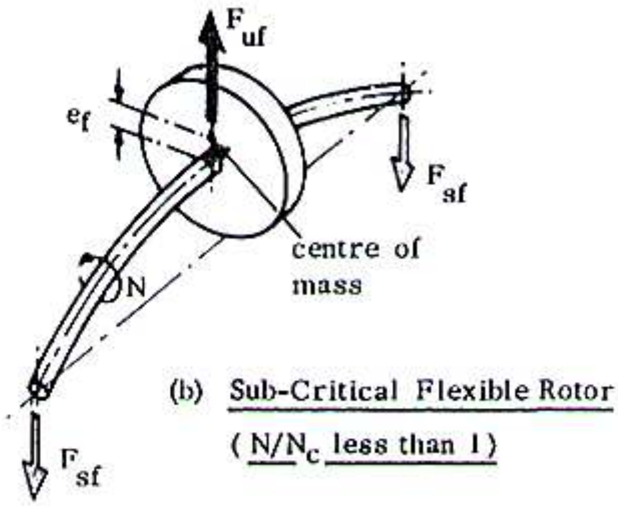

Figure I. 4: Flexible rotor [32].

I.7 Classification of rotors

The classification of rotors has made by the international standard organization (ISO 1973), in order to describe the types of rotors and the quality of balance [31], it may be determinate according to both of geometric and mechanical parameters:

Geomatric parameters	Mechanical parameters
▪ Rotor shaft model (shaft dimensions) ▪ Rotor disc model (disc dimensions) ▪ Bladed rotor model (eg: helicopter) ▪ Free rotor model (the absence of suspension (eg: no bearing))	▪ Rigid rotor model ▪ Jeffcot rotor model (simple rotor) ▪ Real rotor model (flexible rotor with higher speeds)

Table I. 1: Classification of rotors [30].

I.8 Composite materials

Materials have always played a major role in the development and growth of human civilization [33], structural materials can be divided into four basic categories : metals, polymers, ceramics, and composites [34]. In the past decades, the theory of composite materials became one of the most attractive topics in mechanics of solids [35], there has been a major effort to develop composite material systems, and analyze and design structural components made from composite materials [36].

1) Definition

A composite material is different from the conventional macroscopically homogeneous material [37], it is an evolving and growing technical field [38]. Composite material can be defined as a heterogeneous mixture of two or more homogeneous phases which have been bonded together, in order to rectify some shortcoming of a particularly useful component [39], and to achieve a particular function , this combinations may be materials of the same class or of a different class [40].

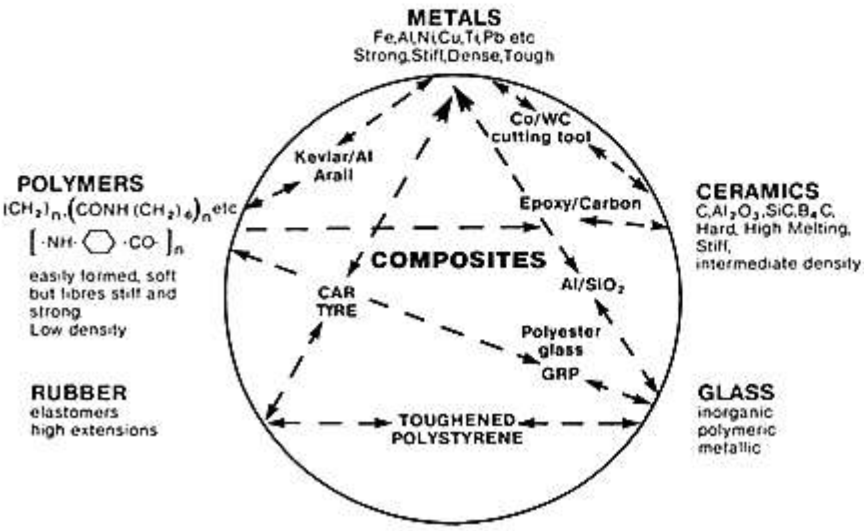

Figure I. 5: The combination of materials on composites [40].

Nowadays, the term advanced composite means specifically this combination of very strong and stiff fibers within a matrix designed to hold the fibers together [39], the matrix keeps the geometric arrangement of fibers and transmits to them the load acting on the composite component [37].

2) The constituents of composite material

In the most general case, a composite material consists of one or more phases discontinuous distributed in a contained phase. The discontinuous phase has mechanical properties superior compared to the continuous one. The continuous phase is called the matrix, and the discontinuous phase is called the reinforcement [41].

A. Matrix

The matrix is the essentially homogeneous material, which is based on polymer metals or ceramics. The choice of matrix is related to the required properties, the intended applications of the composite and the method of manufacture [42].

Its main role is to :

- Bind the reinforcements in such a way as to form a distinct interphase between them [33].
- Transfer the load between fibers and between the composite and the supports.
- Protect the fibers from the environment and mechanical abrasion [43].

The matrix is composed of a resin, which is defined as an organic polymer or prepolymer. This organic matrix may be a thermoset or a thermoplastic [44].

a. Thermoset matrices

A thermoset matrix is formed by the irreversible chemical transformation of a resin system [43], this matrix can be characterized by having polymer chains that become highly cross-linked during cure. Once it is cured, it is in a final rigid configuration and there is nothing that will change it. These matrices are advantageous for high temperature applications of composites [45].

b. Thermoplastic matrices

A thermoplastic matrix does not undergo any chemical transformation during the processing, this matrix can be characterized by having polymer chains that are not cross-linked, moreover, it is softened from the solid state to be processed, and it returns to a solid after the processing is completed. When using these matrices, the operating temperature should be kept below the cure temperature [43] [45].

c. Materials for matrices

By now, matrices are made from polymeric, metal, carbon, and ceramic materials which are classified into two principals groups of materials : organic and mineral.

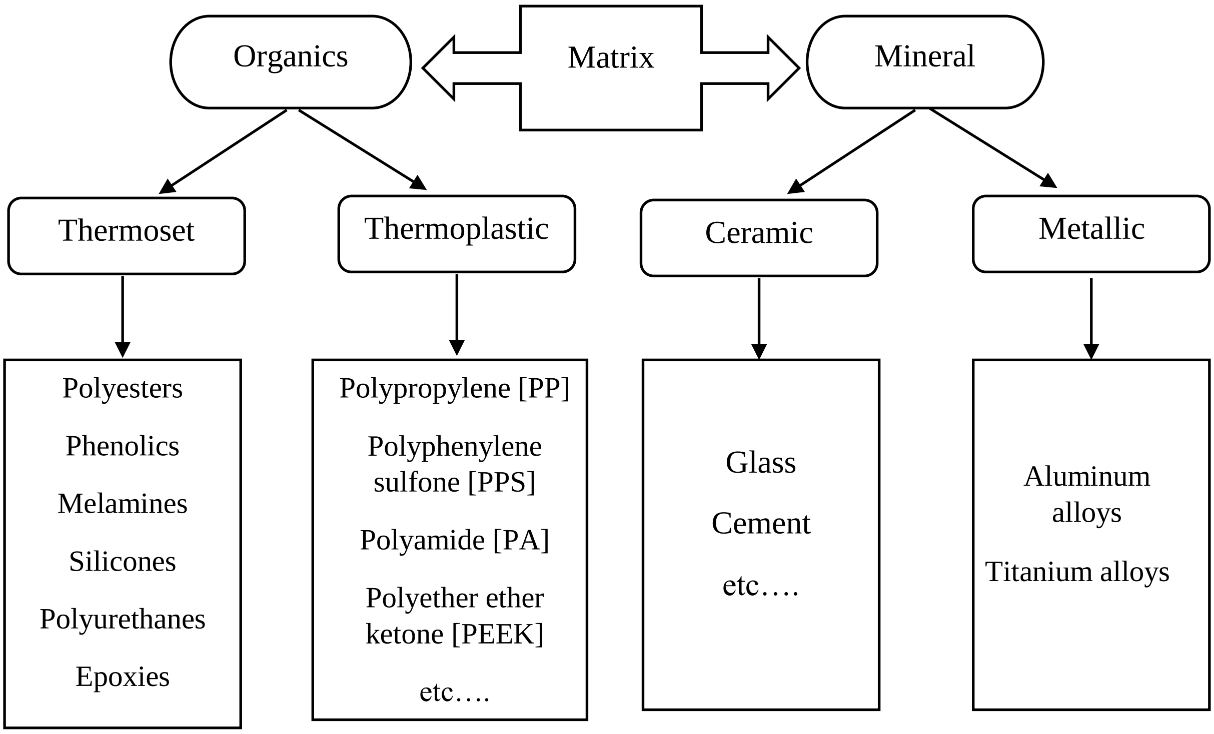

Figure I. 6: The principal materials for the matrix.

B. The fibre

A fiber can be defined as an elongated material having a more or less equiaxial and uniform cross section [39] . They consist of several hundreds or thousands of filaments, each of them having a diameter between 5 and 15 µm, allowing them to be processable on textile machines [37], they can be of different physical forms including : particles, whiskers, short fibers, continuous fibers, and plates [33].

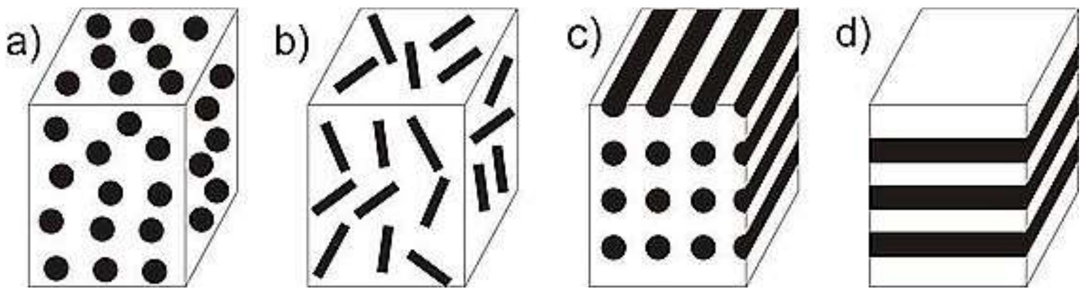

Figure I. 7: The different forms of fibers: a) Particles, b) Short fibers, c) Continuous fibers, d) Plates [45].

In most cases, the reason of using fibers in composites is because they are harder, stronger and stiffer, or in other words, the reinforcement is defined as the strong integral and inert component of a composite that is incorporated into the composite matrix to improve its physical properties [47].

a. Materials for fibres

The reinforcing material is embedded in the matrix material at a macroscopic level [33] major progress continues to be made with inorganic and organic reinforcement materials [47].

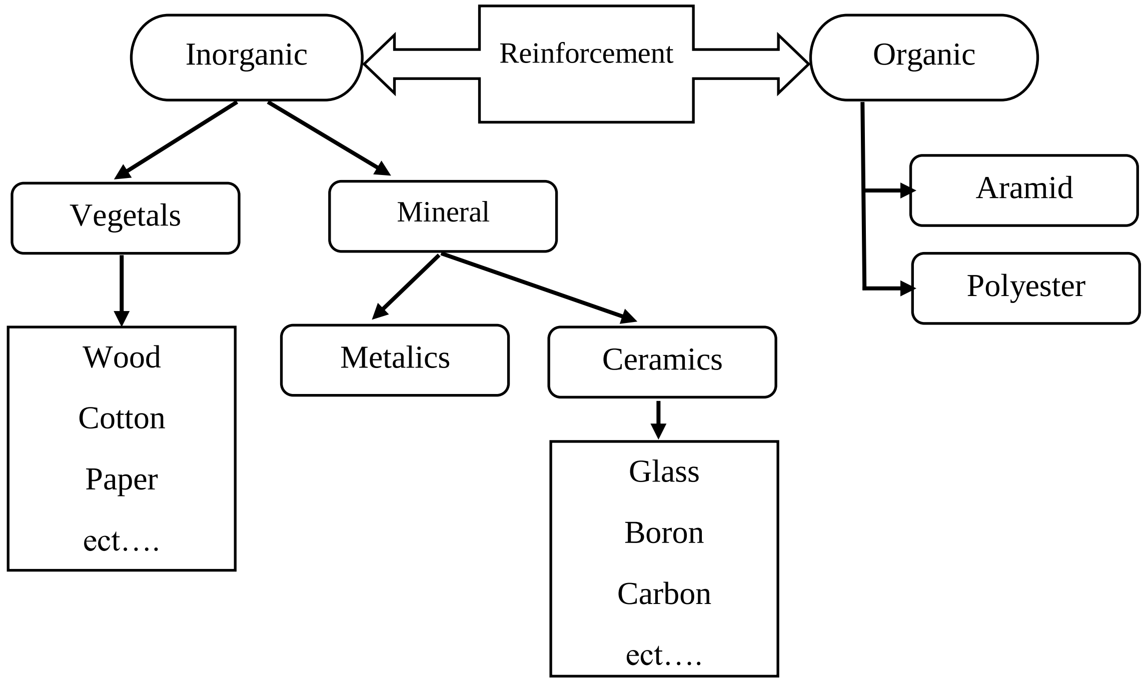

Figure I. 8: The principal materials for the reinforcement.

I.9 Classification and types of composite materials

Classification of composite materials is based on the types of materials of both matrices and fibers, so [33]:

According to the type of matrix material :

- Metal Matrix Composites (MMC).
- Ceramic Matrix Composites (CMC).
- Polymer Matrix Material (PMC).

According to the type of fiber material :

- Particulate Composites.

- Fibrous Composite**s.**

This classification is included into other three groups which are more specific and useful, there is :

- Natural composite Materials
- Micro composite Materials
- Macro composite.

Moreover, the different types of composite are given below in order to get a proper understanding of classification of composites.

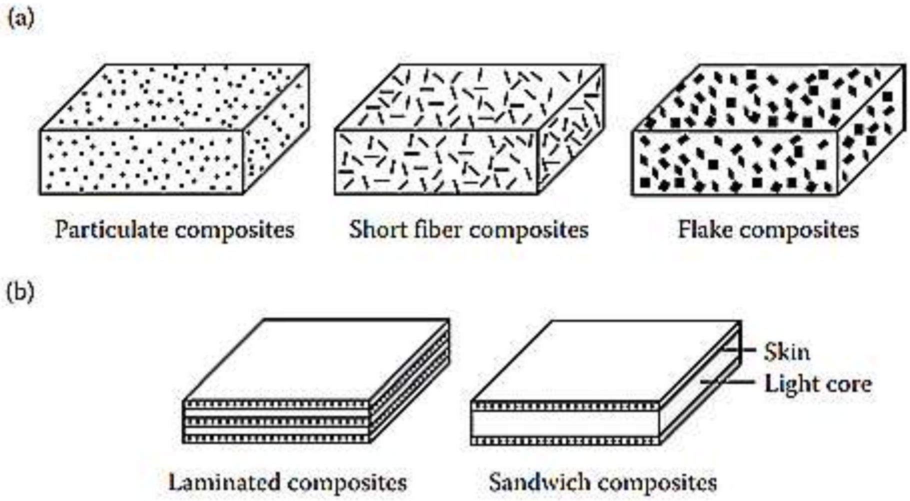

Figure I. 9: Types of composites : (a) phased composites, (b) Layered composites [33].

I.10 Applications of composite materials

Acute interest in investigation and modelling of physical and mechanical properties of composite materials is connected with the constantly increasing area of their applications [35].

Nowadays, composite materials would embrace large sections of : Aerospace & automobile industries, marine construction, renewable energy, modern medicine micro-/nano-material technologies.

In addition, composite materials are currently the hottest topic of researchers in civil and mechanical engineering, chemical engineering, electrical engineering, material science, and solid and structural mechanics.

They are used in many areas of industry and transportation, including land, sea, and air transportation systems, commercial appliances, and electronics and computer systems [48] .

I.11 Advantages and disadvantages of composite materials

The growth of the production of a variety of composite materials and the rapid development of technology and research in this field clearly show advantages of composites [46], furthermore, these lightweight materials have some precise objectives, which cannot be reached with some other conventional materials [49].

Composites proved to be superior to the known homogeneous materials: first of all, they have superior physical and mechanical properties; secondly, it is possible to design the composite structure and to create materials with the prescribed in advance properties optimal for the operation conditions of the whole structure [35].

However, there are other certain disadvantages as well; the table below shows both of advantages and disadvantages of composites:

Advantages	Disadvantages
1. High tensile strength and stiffness	1. Low service temperature
2. High specific strength and specific stiffness	2. Sensitivity to radiation and moisture
3. High fatigue strength	3. Low elastic properties in the transverse direction
4. Inherent material damping and good impact properties	4. Complex design and analysis
5. Tailorable properties	5. Complex mechanical characterization
6. Design flexibility	6. High cost of raw materials and fabrication
7. Less corrosion	7. Difficulty in jointing
8. Simple manufacturing techniques	
9. Near net shape part and lower part count	
10. Cost-effective product development	

Table I. 2: The advantages and disadvantages of composite materials [33].

I.12 Dynamic behavior analysis of composite rotor

The rotors are considered as the most important part of machines, and, composite materials are widely used in the design of rotors, and this is why the dynamic behavior of composite rotors becomes a subject worthily of study. The interest studies and research in the field of composites for rotor dynamics, numerically and experimentally has been demonstrated, and, it makes them improve a large number of designs in rotor dynamics.

As a definition, the rotor dynamic is the study of the stability; it plays a major role in improving the security and performance of the system [50].

Researchers worked on the developing of a composite shaft, it is because of: rotor dynamic modelling of a rotating composite shaft is necessary, and the shafts made from composites are mostly used in almost all the applications.

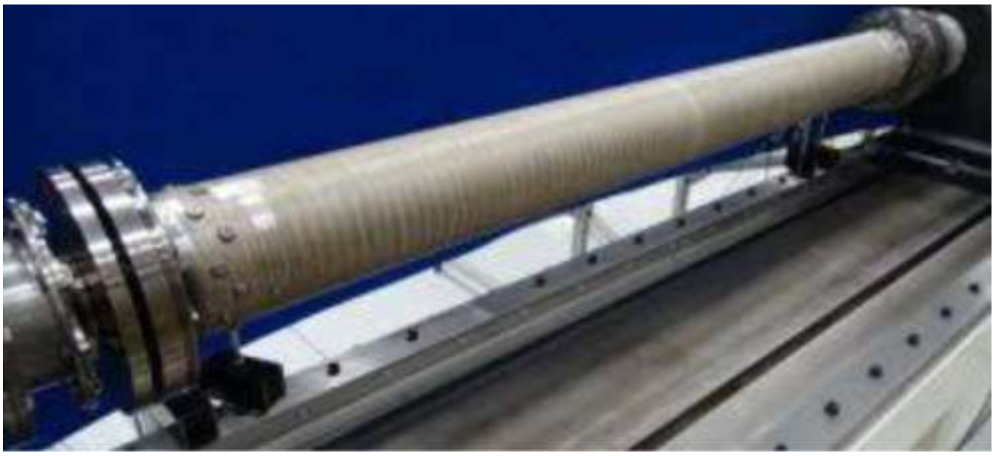

Figure I. 10: Composite driveshaft [24].

The dynamic behavior of the rotating structure demands: the structure to be at rest and, it is supposed to be cyclically symmetric. The equivalent approach for modulus beam is usually used in the studies of the behavior of composite rotors [51].

Establishing of methodologies of vibration is the purpose of rotor dynamics, and, the objective of the study of the dynamic behavior of rotors is as following [50]:

- Predict critical speeds.
- Determine design changes to change critical speeds.
- Predict the natural frequencies of vibration in torsion, bending and coupling.
- Predict amplitudes of synchronous vibrations caused by the imbalance of the rotor.
- Determine design changes to remove dynamic instability.

It is observed that the conventional rotor have dynamic parameters as well as critical speeds, natural frequencies, damping factor, imbalance of response and stability threshold, which they have been analyzed in detail, plus, there is a theory that the formulation of the dynamic of rotors is based on, it is the beam theory [23].

I.13 **Conclusion**

The major facts and properties of the composite materials and the rotor dynamics has been identified and explained in a short way, which leads to the well-known what this study is based on, and it clearly appears that the use of the composite materials made the shaft system improve their performance, furthermore, and for a general-purpose, the composite-material shafts have been sought as new potential candidates for replacement of the conventional shaft in many application areas. Also, and thanks to their high performance, modeling the mechanical behavior of composite materials becomes necessary. The composite shaft modeling using beam theory and finite element model will be discuss in the next chapter.

<u>CHAPTER II</u>

Mathematical and Finite Element Models

II.1 Introduction

The mathematical model of the rotor is derived from the Lagrange's equation which is obtained from the strain and kinetic energies expressions. The mathematical representation of the specific physical phenomena that involve rotating machines requires a reliable design tool.

The main phenomena that occur in rotor dynamics can be evaluated by the finite element method. This chapter focuses exclusively on the finite element model based upon the Timoshenko beam theory to obtain matrix equations. In this context, the present work is dedicated to a numerical investigation the unbalance response (dynamic behaviour) of a rotating composite shaft supported by flexible journal bearings.

II.2 Beam theory

The term "Beam" can be defined as a "bar" when the external forces are axial, and as "shaft" when it is subjected to torsion or when it has a rotational movement.

The different studies on the beam theory are based on either the equivalent modulus beam theory (EMBT) or layer wise beam theory (LBT), furthermore, the rotodynamic formulations are based on those two theories. The beam theory is perhaps the most successful theory in all of structural analysis.

The idea behind beam theory based on three different beam models which may be used in the finite element analysis, and they are:

- Euler-Bernoulli beam.
- Rayleigh beam
- Timoshenko beam.

In Euler-Bernoulli beam theory, only pure bending energy is considered. In case of Rayleigh beam theory, in addition of transitional kinetic energy cross-sectional rotational kinetic energy is considered which is called rotary inertia. In Timoshenko beam theory, shear strain energy is taken into account [51] .

The Timoshenko beam theory formulation based on an equivalent modulus which has been developed and extended for a composite rotor [52] , and it is known that the equivalent modulus beam theory (EMBT) is widely used to study the dynamic behavior of the composite shaft, so, in the present analysis, the Timoshenko beam theory will be used for the development of a governing equation of the continuous system analysis [25].

II.3 Kinematic equations

Now, the focus will be on the governing equations, by using the Timoshenko beam theory, and in the layer "n" of the cross section of the beam element, the continuous displacement field at material points along the shaft cross section is described as [53]:

$$\left\{u(x,y,z,t)\right\} = \begin{cases} u_x(x,y,z,t) = -y\theta_z(x,t) + z\theta_y(x,t) \\ u_y(x,y,z,t) = v(x,t) \\ u_z(x,y,z,t) = w(x,t) \end{cases} \tag{2.01}$$

Where:

- "*ux*", "*uy*" and "*uz*": are the displacements of a generic point of the cross section along *x*, *y* and *z* directions respectively.
- "*v* (*x; t*)" and "*w* (*x; t*)": denote respectively the flexural displacement in *y* and *z* directions of the point on the reference axis of the shaft.
- "*θy* (*x; t*)" and "*θz* (*x; t*)": are respectively the rotation angles of the cross section about *y* and *z* axis.

1) Strain – displacement relation

The strain – displacement relation based on the above displacement assumption can be represented by the following equation [53]:

$$\{\varepsilon\} = \begin{cases} \varepsilon_{xx} = -y\dfrac{\partial\theta_z}{\partial x} + z\dfrac{\partial\theta_y}{\partial x} \\ \gamma_{xy} = -\theta_z + \dfrac{\partial v}{\partial x} \\ \gamma_{xz} = \theta_y + \dfrac{\partial w}{\partial x} \\ \varepsilon_{yy} = \varepsilon_{zz} = \gamma_{yz} = 0 \end{cases} \tag{2.02}$$

The strain components in cylindrical coordinate system can be expressed in terms of their counter parts in the Cartesian coordinate system as [53]:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\phi\phi} \\ \varepsilon_{rr} \\ \gamma_{x\phi} \\ \gamma_{r\phi} \\ \gamma_{xr} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sin^2\phi & \cos^2\phi & 0 & -2\cos\phi\sin\phi & 0 \\ 0 & \cos^2\phi & \sin^2\phi & 0 & 2\cos\phi\sin\phi & 0 \\ 0 & 0 & 0 & -\sin\phi & 0 & \cos\phi \\ 0 & -\cos\phi\sin\phi & \cos\phi\sin\phi & 0 & (\cos\phi^2-\sin\phi^2) & 0 \\ 0 & 0 & 0 & \cos\phi & 0 & \sin\phi \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

(2.0 3)

Since: $\varepsilon_{yy} = \varepsilon_{zz} = \gamma_{yz} = 0$, the strain components in the cylindrical coordinate system can be written as [53]:

$$\{\varepsilon\} = \begin{cases} \varepsilon_{xx} = \varepsilon_{xx} \\ \gamma_{x\phi} = -\gamma_{xy}\sin\phi + \gamma_{xz}\cos\phi \\ \gamma_{xr} = \gamma_{xy}\cos\phi + \gamma_{xz}\sin\phi \end{cases} \tag{2.04}$$

Therefore, the tensor of the strains in the layer “n” at a point “P” according to the cylindrical coordinates is [53]:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{x\phi} \\ \gamma_{xr} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\sin\phi & \cos\phi \\ 0 & \cos\phi & \sin\phi \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xy} \\ \gamma_{xz} \end{Bmatrix} \tag{2.05}$$

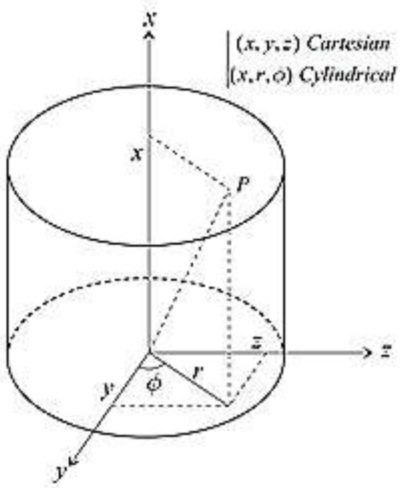

Figure II. 1: Transformation to cylindrical coordinate system (x,r,φ) [53].

2) Stress – strain relation

The linear stress – strain relation of a structure is given by Hooke's law and it can be established:

$$\{\sigma\}=[E]\{\varepsilon\} \tag{2.06}$$

With: (σ) and (ε) are the stress and the strain respectively. The generalized Hooke's law for an orthotropic material is written as follow :

$$\{\sigma\}=[C]\{\varepsilon\}, or:\{\varepsilon\}=[S]\{\sigma\} \tag{2.07}$$

While: [C] and [S] can be defined as the stiffness matrix and the compliance matrix respectively.

Only the expression of the stiffness matrix will be developed here, the compliance matrix can be obtained by considering that $[S]=[C]^{-1}$.When linked to the orthotropic axis, Hooke's law takes the following form [23]:

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{Bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{Bmatrix} \tag{2.08}$$

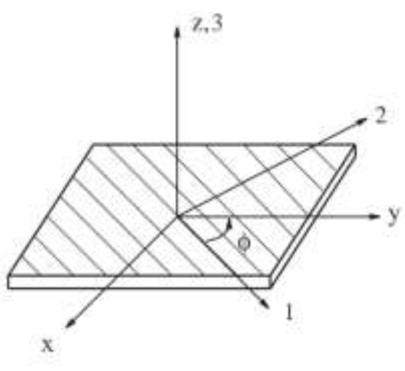

Figure II. 2: Plan of ply [54].

Where: "1*;* 2*;* 3" are the orthotropic axes. "1" is the fiber direction, "2" is the direction transversal to the fibers in the ply, "3" is the direction perpendicular to the ply, and, "φ" is the ply fiber angle.

Since the shapes of the cross-sections of the composite shaft are assumed circular, it is more convenient to express the stress–strain relations of the composite material of the shaft using the cylindrical coordinate system ($\vec{i}$; $\vec{e_r}$; $\vec{e_\theta}$) [56]. The stress – strain relation components in this coordinates system is given as:

$$\{\sigma\}=[C']\{\varepsilon\}=[R]^{-1}[C][R]^{-T}\{\varepsilon\} \tag{2.09}$$

Where: $[C']$ is the stiffness matrix related to the cylindrical system axis.

$[R]$ is the basic change matrix of the stresses expressed in function of the angle "η" called the orientation of fibers [23]:

$$[R]=\begin{bmatrix} \cos^2\eta & \sin^2\eta & 0 & 0 & 0 & 2\cos\eta\sin\eta \\ \sin^2\eta & \cos^2\eta & 0 & 0 & 0 & -2\cos\eta\sin\eta \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\eta & -\sin\eta & 0 \\ 0 & 0 & 0 & \sin\eta & \cos\eta & 0 \\ -2\cos\eta\sin\eta & 2\cos\eta\sin\eta & 0 & 0 & 0 & \cos^2\eta-\sin^2\eta \end{bmatrix} \tag{2.10}$$

Well, in the cylindrical coordinate system (x; r; θ), the stresses are expressed according to the following relation [23]:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{\phi\phi} \\ \sigma_{rr} \\ \tau_{r\phi} \\ \tau_{xr} \\ \tau_{x\phi} \end{Bmatrix}=\begin{bmatrix} C'_{11} & C'_{12} & C'_{13} & 0 & 0 & C'_{16} \\ C'_{12} & C'_{22} & C'_{23} & 0 & 0 & C'_{26} \\ C'_{13} & C'_{23} & C'_{33} & 0 & 0 & C'_{36} \\ 0 & 0 & 0 & C'_{44} & C'_{45} & 0 \\ 0 & 0 & 0 & C'_{45} & C'_{55} & 0 \\ C'_{16} & C'_{26} & C'_{36} & 0 & 0 & C'_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\phi\phi} \\ \varepsilon_{rr} \\ \gamma_{r\phi} \\ \gamma_{xr} \\ \gamma_{x\phi} \end{Bmatrix} \tag{2.11}$$

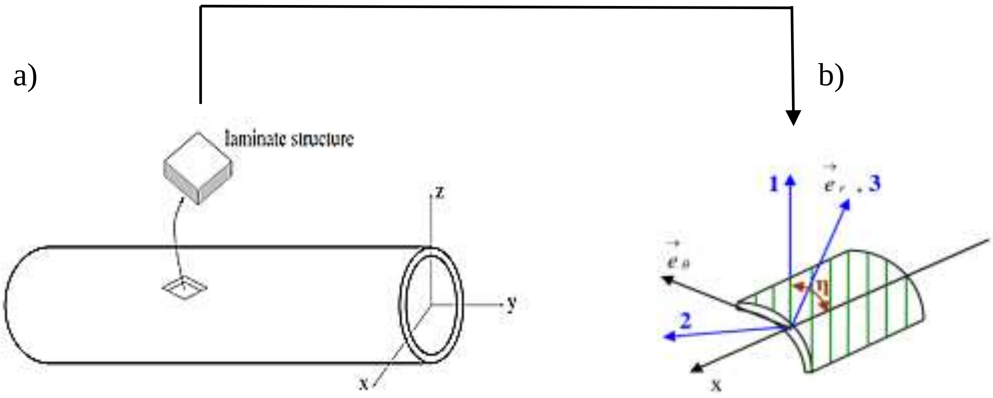

Figure II. 3: a) Composite rotor [54]. b) Main axes (1,2,3) and reference axes (x; er; eθ) of a layer "n" [23].

Moreover, the previous relation "relation (2.11) is written in simplified form as follows:

$$\begin{Bmatrix} \sigma_{xx} \\ \tau_{xr} \\ \tau_{x\theta} \end{Bmatrix} = \begin{bmatrix} C'_{11} & 0 & C'_{16} \\ 0 & C'_{55} & 0 \\ C'_{16} & 0 & C'_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xr} \\ \gamma_{x\theta} \end{Bmatrix} \tag{2.12}$$

From here, it is evident that there are equations which can be extracted from this new relation "relation (2.12)":

$$\begin{aligned} \sigma_{xx} &= C'_{11}\varepsilon_{xx} + k_s C'_{16}\gamma_{x\theta} \\ \tau_{xr} &= k_s C'_{55}\gamma_{xr} \\ \tau_{x\theta} &= k_s C'_{16}\varepsilon_{xx} + k_s C'_{66}\gamma_{x\theta} \end{aligned} \tag{2.13}$$

Where: "ks" is the shear correction factor, and as it is known that "$\sigma_{\theta\theta} = \sigma_{rr} = \sigma_{r\theta} = 0$" and "$\tau_{xr} = \tau_{rx}$; $\tau_{x\theta} = \tau_{\theta x}$", the tensor of the stresses in the layer "n" at a point "P" according to the cylindrical coordinates is:

$$\{\sigma\} = \begin{bmatrix} \sigma_{xx} & \tau_{xr} & \tau_{x\theta} \\ \tau_{rx} & 0 & 0 \\ \tau_{\theta x} & 0 & 0 \end{bmatrix} \tag{2.14}$$

In the main axes, the stiffness constants reduced in function of the elasticity moduli are [23]:

$$C_{11}=\frac{E_L}{1-\nu_{LT}\nu_{TL}}=\frac{E_L}{1-\frac{E_T}{E_L}\nu^2{}_{LT}} \quad , \quad C_{12}=\nu_{LT}C_{22}$$

$$C_{22}=\frac{E_T}{E_L}C_{11} \quad , \quad C_{44}=G_{TP} \tag{2.15}$$

$$C_{55}=G_{LP} \quad , \quad C_{66}=G_{LT}$$

With: "$\nu_{LT}=\nu_{12}$","$\nu_{LP}=\nu_{13}$","$\nu_{TP}=\nu_{23}$", and:

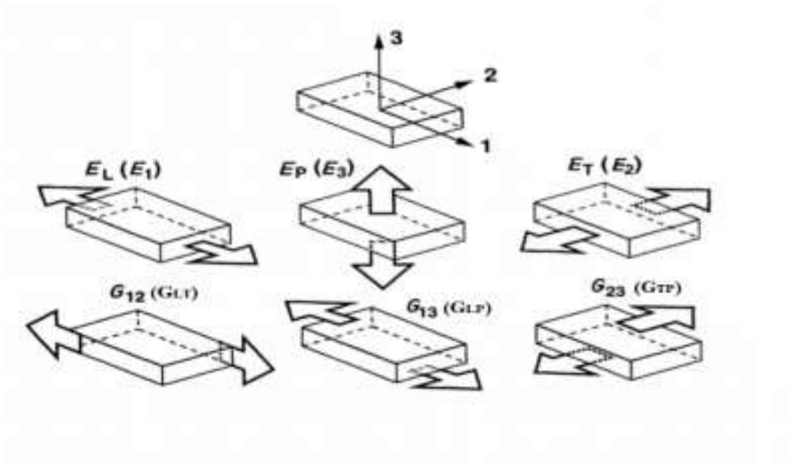

Figure II. 4: Definition of moduli for orthotropic materials [40]

The transverse plane marked by the directions 2 and 3 are equivalent, so, the following parameters have to be identified for each ply:

- *E*1, G12 and ν12 : are Young's modulus shear modulus and poison ratio respectively in the longitudinal direction
- *E*2, G23 and ν23 : are Young's modulus shear modulus and poison ratio respectively in the transversal direction.

Otherwise, in the cylindrical system, the stiffness coefficients are in function of the coefficients referred to the layer axes, and they are expressed by the angle "η" in the main direction (1 or L) as follows [23]:

$$\begin{aligned}
C'_{11}&=C_{11}\cos^4\eta+C_{22}\sin\eta+2(C_{12}+2C_{66})\sin^2\eta\cos^2\eta\\
C'_{16}&=(C_{11}-C_{12}-2C_{66})\sin\eta\cos^3\eta+(C_{12}-C_{22}+2C_{66})\sin^3\eta\cos\eta\\
C'_{66}&=[C_{11}+C_{22}-2(C_{12}+C_{66})]\sin^2\eta\cos^2\eta+C_{66}(\sin^4\eta+\cos^4\eta)\\
C'_{55}&=C_{44}\sin^2\eta+C_{55}\cos^2\eta
\end{aligned} \tag{2.16}$$

II.4 Energy expressions

While in operation, rotors of machines have a great deal of rotational energy, and a small amount of vibrational energy [56], in order to study the dynamics of a system comprising one or more rotors, it is possible to write the equations of motion either in a fixed reference frame or in a reference frame rotating at the same speed as the rotor [50].

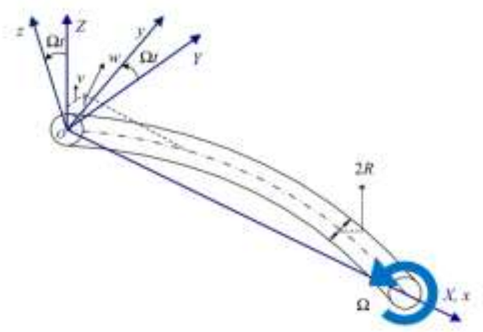

Figure II. 5: A micro-rotating beam with circular cross section and the corresponding coordinate system [57].

The “XYZ” triad is a fixed frame of reference and “xyz” triad is a rotating frame of reference with “X” and “x” being collinear and coincident with the undeformed rotor center line. Rotating frame is one which rotates about the longitudinal axes at angular Ω [58].

A typical cross section of the rotor in a deformed state is defined relative to XYZ by the translations “v(x,t)” and “w(x,t)” in the Y and Z directions respectively to locate the elastic centreline and small angle rotations θy(s,t) and θz(s,t) about Y and Z respectively to orient the plane of the cross-section. The “xyz” triad is attached to the cross-section with the "x" axis normal to the cross-section [58].

The angular velocity vector of the rotational rigid-body motion of the element can be determined by considering three successive Euler angles : θx(x,t) , θy(x,t) and θz(x,t), where

θx(x,t)=Ωt because the torsional deformation of the element has been assumed to be negligible. In terms of the Euler angles, the angular velocity vector of a given element is expressed as [57]:

$$\omega = \omega_1 e_1 + \omega_2 e_2 + \omega_3 e_3 = \left(\Omega - \dot{\theta}_z \sin\theta_y\right) e_1 + \left(\dot{\theta}_z \cos\theta_y \sin\Omega t + \dot{\theta}_y \cos\Omega t\right) e_2 + \left(\dot{\theta}_z \cos\theta_y \cos\Omega t - \dot{\theta}_y \sin\Omega t\right) e_3 \tag{2.17}$$

Where : e_1, e_2, e_3 are the unit base vectors of the coordinate system X3–Y3–Z3, which is the one completely attached to the element.

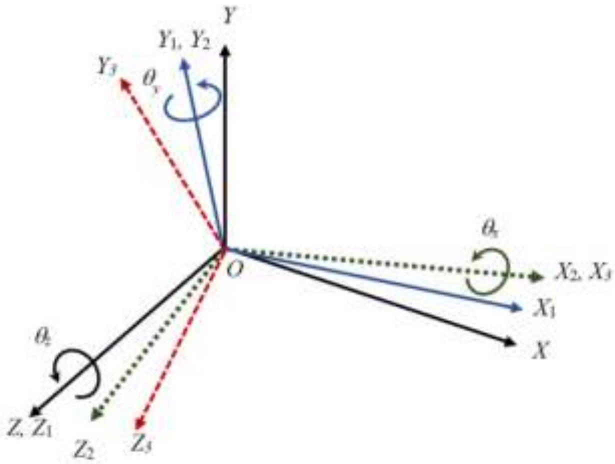

Figure II. 6: Three-axis Euler angles rotations [57]**.**

The general equations of the rotor are obtained from the expressions of the energies of each element of the rotor.

1) **The kinetic energy of the shaft, the disk and the unbalance**

The expressions of kinetic energies are necessary to characterize the shaft ,the disc and the unbalance [59], either a composite rotor consisting of "N" layer of orthotropic material, and according to the beam theory , for each ply of beam element of length "L" and constant cross-section [27] , the kinetic energy is given by the form :

For an element of the shaft [53]:

$$T_s = \frac{1}{2}\int_0^L \left[I_m \left(\dot{v}^2 + \dot{w}^2\right) + I_d \left(\dot{\theta}_y^2 + \dot{\theta}_z^2\right) - 2\Omega I_p \theta_y \dot{\theta}_z + I_p \Omega^2 \right] dx \tag{2.18}$$

With:

$$\begin{aligned} I_m &= \pi \sum_{n=1}^{k} \rho_n \left(R_n^2 - R_{n-1}^2\right) \\ I_d &= \frac{\pi}{4} \sum_{n=1}^{k} \rho_n \left(R_n^4 - R_{n-1}^4\right) \\ I_p &= \frac{\pi}{2} \sum_{n=1}^{k} \rho_n \left(R_n^4 - R_{n-1}^4\right) \end{aligned} \tag{2.19}$$

Where:

- $2\Omega I_p \theta_y \dot{\theta}_z$: this term represents the gyroscopic effect.
- $I_d\left(\dot{\theta}_y^2+\dot{\theta}_z^2\right)$: this one represents the effect of rotary inertia.
- "I_m": the moment of mass inertia. "I_d": the moment of diametrical inertia. "I_p": the moment polar inertia.

For an element of the disk [53]: "The disk is supposed rigid, so, only its kinetic energy is considered, plus, both of the translational and the rotational energies are summed in the following form":

$$T_d = \frac{1}{2} m_d \left(\dot{v}^2+\dot{w}^2\right)+\frac{1}{2} I_{dx}\left(\dot{\theta}_z^2+\dot{\theta}_y^2\right)+\frac{1}{2} I_{dy}\Omega^2+I_{dy}\Omega\dot{\theta}_z\theta_y \tag{2.20}$$

With:

- $\frac{1}{2} m_d \left(\dot{v}^2+\dot{w}^2\right)$: Kinetic energy of an element in translation in a plane.
- $\frac{1}{2} I_{dx}\left(\dot{\theta}_z^2+\dot{\theta}_y^2\right)$: Kinetic energy of rotation of an element about the "y" and "z" axis.
- $\frac{1}{2} I_{dy}\Omega^2$: Constant term for disc rotation energy.
- $I_{dy}\Omega\dot{\theta}_z\theta_y$: Gyroscopic effect (Coriolis).

For the unbalance [50]: "The unbalance is defined by a mass "mu" situated at a distance "d" from the geometric center of the shaft. The unbalance mass is negligible in the relation of the rotor mass."

$$T_u = m_u \Omega d\left(\dot{u}\sin\Omega t-\dot{w}\cos\Omega t\right) \tag{2.21}$$

2) The strain energy of the shaft

The deformation energy is not affected by the movement of the support because it only depends on the stresses and therefore on the deformation of the shaft in relation to the support [30].

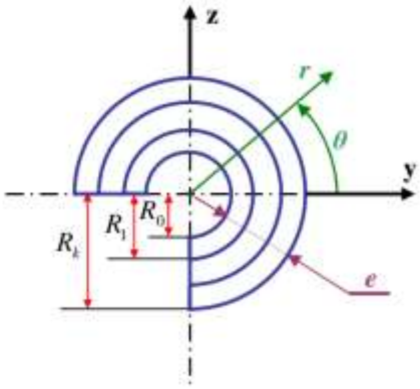

Figure II. 7: “k” shaft layers in composite materials [23]**.**

With: “R0” is the inner radius of the shaft, “Rk” is the outer radius of the shaft and “e” is the thickness of the shaft.

The expression for the strain energy of the shaft is :

$$\begin{aligned} U &= \frac{1}{2}\int_V \{\sigma_{ij}\}^t \{\varepsilon_{ij}\} dV \\ &= \frac{1}{2}\int_V (\sigma_{xx}\varepsilon_{xx} + \sigma_{rr}\varepsilon_{rr} + \sigma_{\phi\phi}\varepsilon_{\phi\phi} + \tau_{xr}\gamma_{xr} + \tau_{x\phi}\gamma_{x\phi} + \tau_{r\phi}\gamma_{r\phi})\, dV \end{aligned} \tag{2.22}$$

Where: “V” is the volume of the shaft, and, since: $\varepsilon_{\phi\phi} = \varepsilon_{rr} = \gamma_{r\phi} = 0$ the deformation expression becomes [54]:

$$U = \frac{1}{2}\int_V (\sigma_{xx}\varepsilon_{xx} + \tau_{xr}\gamma_{xr} + \tau_{x\phi}\gamma_{x\phi})\, dV \tag{2.23}$$

Using the equation (2.13), the deformation energy expression becomes:

$$U = \frac{1}{2}\int_V (C'_{11}\varepsilon_{xx}^2 + 2kC'_{16}\gamma_{x\phi}\varepsilon_{xx} + kC'_{55}\gamma_{xr}^2 + kC'_{66}\gamma_{x\phi}^2)\, dV \tag{2.24}$$

Where: $2kC'_{16}\gamma_{x\phi}\varepsilon_{xx}$ accounts for the shear-normal coupling effect.

Replacing the relations for the cross section rotation where $y = r \cos \varphi$ and $z = r \sin \varphi$, and integrating over the shaft cross sectional area by summing up the contribution of each orthotropic layer, the deformation energy in expanded form is given by [54] :

$$U=\frac{1}{2}A_{11}\int_0^L\left[\left(\frac{\partial\theta_z}{\partial x}\right)^2+\left(\frac{\partial\theta_y}{\partial x}\right)^2\right]dx+\frac{1}{2}k_sA_{16}\int_0^L\left[-\theta_y\frac{\partial\theta_z}{\partial x}-\frac{\partial\theta_z}{\partial x}\frac{\partial w}{\partial x}+\theta_z\frac{\partial\theta_y}{\partial x}-\frac{\partial\theta_y}{\partial x}\frac{\partial v}{\partial x}\right]dx+$$
$$\frac{1}{2}k_s\left(A_{55}+A_{66}\right)\int_0^L\left[\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2+\theta_z^2+\theta_y^2+2\theta_y\frac{\partial w}{\partial x}-2\theta_z\frac{\partial v}{\partial x}\right]dx$$

(2.25)

With:

$$\begin{aligned}A_{11}&=\frac{\pi}{4}\sum_{n=1}^{k}C'_{11n}\left(R_n^4-R_{n-1}^4\right)\\A_{16}&=\frac{2\pi}{3}\sum_{n=1}^{k}C'_{16n}\left(R_n^3-R_{n-1}^3\right)\\A_{55}&=\frac{\pi}{2}\sum_{n=1}^{k}C'_{55n}\left(R_n^2-R_{n-1}^2\right)\\A_{66}&=\frac{\pi}{2}\sum_{n=1}^{k}C'_{66n}\left(R_n^2-R_{n-1}^2\right)\end{aligned} \tag{2.26}$$

Where: “k” is the number of layers and “n” is the index of layers.

3) The virtual work of the bearings

In general, bearings which induce external forces acting on the shaft have stiffness and damping characteristics. These characteristics are on the cross-sectional plan according to the directions shown in the following figure:

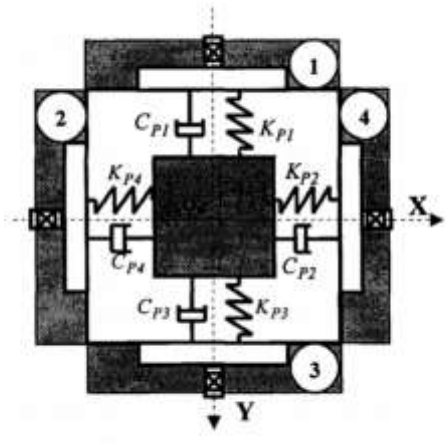

Figure II. 8: Damping and stiffness of bearing [60].

The virtual work of the external forces acting on the shaft has the following expression [50]:

$$\delta W = -k_{xx} u \delta u - k_{zz} w \delta w - D_{xx} \dot{u} \delta u - D_{zz} \dot{w} \delta w \tag{2.27}$$

And, the virtual work of the forces acting on the shaft is written as follows:

$$\delta W = F_u \delta u + F_w \delta w \tag{2.28}$$

Where: “Fu and Fw” are the generalized forces.

So, according to the two previous equations, these generalized forces can be represented in matrix form as follows [50]:

$$\begin{bmatrix} F_u \\ F_w \end{bmatrix} = -\begin{bmatrix} k_{xx} & 0 \\ 0 & k_{zz} \end{bmatrix}\begin{bmatrix} u \\ w \end{bmatrix} - \begin{bmatrix} D_{xx} & 0 \\ 0 & D_{zz} \end{bmatrix}\begin{bmatrix} \dot{u} \\ \dot{w} \end{bmatrix} \tag{2.29}$$

Where:

- $k_{xx}, k_{zz}, D_{xx}, D_{zz}$ **:** are stiffnesses and damping according to the “x” and “z” directions of the shaft.

II.5 Finite Element Model

There are two methods which are often used for the dynamic analysis of rotors: “The transfer matrix method (TMM)” and “ The finite element method (FEM)”, both of these methods have been used extensively for modeling and analyses of rotor systems, but, the finite element method is known among the most important and efficient methods for modelling and solving complex problems in engineering sciences, especially in rotor dynamics [30] [25].

A finite element model of the rotor was created using two noded Timoshenko beam elements with gyroscopic effects included [25]. It has been seen that the finite element method consists mainly in setting out the matrices and solving the equations of motion [57], it is applied to the solution of the governing partial differential equation of the Timoshenko beam with rotary and gyroscopic effects. Elemental matrices are presented for the mass, stiffness, and gyroscopic effect [25].

II.6 Equation of motion

The general equation of motion is obtained by applying Lagrange's equations on the energy expressions of the rotor element by outputting the different characteristic matrices of

the system (mass matrix, stiffness matrix and damping matrix), the Lagrange's equation is given by [30]:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U}{\partial q_i}=F_{q_i} \tag{2.30}$$

Where: " q_i ": Independent generalized coordinates. With: "i" is the number of degrees of freedom. And " F_{q_i} ": The generalized forces vector.

The anisotropic properties of composite materials and their lightness can be used to optimize composite shafts in order to improve their dynamic behavior, the differential equation that represents the dynamic behavior of a composite flexible rotor system is as follows [61]:

$$[M]\{\ddot{q}\}+[D+\Omega G]\{\dot{q}\}+[K]\{q\}=\{W+F_u\} \tag{2.31}$$

Where:

- [M]: Mass matrix. [D]: Damping matrix. [K]: Stiffness matrix. [G]: Gyroscopic effect.
- Ω : The shaft rotation speed. $\{W\}$: The weight of the rotating parts. $\{F_u\}$: The unbalance forces, and, the vector $\{q\}$ contains the generalized displacements.

II.7 Elemental matrices

The product of the three finite element matrices (mass, stiffness and gyroscopic matrices) is based on introducing the shape function in the expressions of energy, and then, application of the equations of Lagrange [27]. The elemental matrices are therefore of order 8, because they represent conventional four degrees of freedom, two translational and two rotational for all particles in the structure [30].

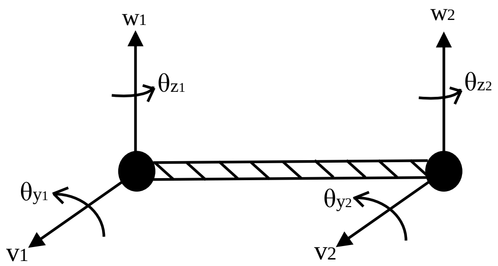

Figure II. 9: Finite element model of beam element.

1) For an element of the disk

The disc is modelled by node with four degrees of freedom: two rotational displacements "θy" and "θz" around the "y" and "z" axes, and two translational displacements "v" and "w" along the "Oy and Oz" axes [62].

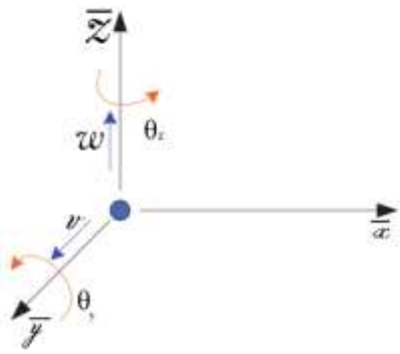

Figure II. 10: Finite element model of the disk [62].

Furthermore, the nodal displacement field is given by: $q=\begin{Bmatrix} v \\ w \\ \theta_y \\ \theta_z \end{Bmatrix}$, and by applying the Lagrange's equation on the kinetic energy expression of the disk (equation 2.20), the equation becomes [62]:

$$\frac{d}{dt}\left(\frac{\partial T_D}{\partial \dot{q}}\right)-\frac{\partial T_D}{\partial q}=[M_d]\{\ddot{q}\}+\Omega[G_d]\{\dot{q}\} \tag{2.32}$$

With:

$$[M_d]=\begin{bmatrix} M_D & 0 & 0 & 0 \\ 0 & M_D & 0 & 0 \\ 0 & 0 & I_{Dy} & 0 \\ 0 & 0 & 0 & I_{Dy} \end{bmatrix} \tag{2.33}$$

$$[G_d]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_{Dy} \\ 0 & 0 & I_{Dy} & 0 \end{bmatrix} \tag{2.34}$$

Where: $[M_d]$: is the mass matrix of the disk, and: $[G_d]$: is the gyroscopic matrix of the disk [62].

2) For an element of bearing

The bearings are modelled by considering the shaft as a linear viscoelastic solid. The vector displacements are that which corresponds to the four degrees of freedom of the node of the section where the bearing is located. The effect of the elastic forces and damping which appear in the bearings is taken into account by the following matrices [24]:

$$\begin{pmatrix} F_v \\ F_w \end{pmatrix}=-\begin{pmatrix} k_{yy} & 0 \\ 0 & k_{zz} \end{pmatrix}\begin{pmatrix} v \\ w \end{pmatrix}-\begin{pmatrix} D_{yy} & 0 \\ 0 & D_{zz} \end{pmatrix}\begin{pmatrix} \dot{v} \\ \dot{w} \end{pmatrix} \tag{2.35}$$

Where: the first matrix is a stiffness matrix and the second is viscous damping matrix.

3) For an element of unbalance

As it is known that the unbalance is characterize by its own mass, and by applying the Lagrange's equation on the kinetic energy expression of the unbalance, the new equation is given by [62]:

$$\frac{d}{dt}\left(\frac{\partial T_u}{\partial \dot{q}}\right)-\frac{\partial T_u}{\partial q}=-m_u\Omega^2 d\begin{Bmatrix} \sin(\Omega t) \\ \cos(\Omega t) \end{Bmatrix} \tag{2.36}$$

With: $q=\begin{Bmatrix} v \\ w \end{Bmatrix}$

4) For an element of shaft

The shaft is modelled by two nodes, each one with four degrees of freedom: two rotational displacements “θyi” and “θzi” around the “y” and “z” axes, and two translational displacements “vi” and “wi” along the “y” and “z” axes [62].

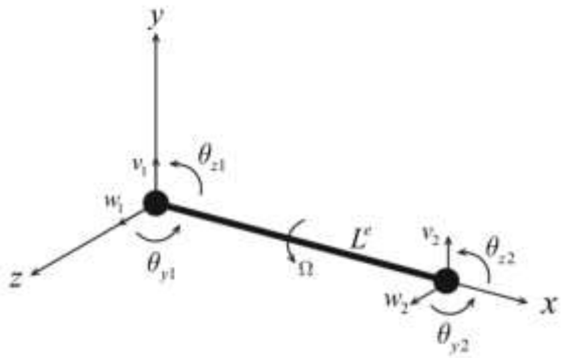

Figure II. 11: Finite element model of the shaft[54]**.**

Since the shaft is represented by two nodes, each one with four degrees of freedom, so the shaft has 8 degrees of freedom, therefore, it indicates that the nodal displacement field is written as follows [62]:

$$\{q\}=\begin{Bmatrix} q_v=\begin{Bmatrix} v \\ \theta_z \end{Bmatrix}=\begin{Bmatrix} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \end{Bmatrix} \\ q_w=\begin{Bmatrix} w \\ \theta_y \end{Bmatrix}=\begin{Bmatrix} w_1 \\ \theta_{y1} \\ w_2 \\ \theta_{y2} \end{Bmatrix} \end{Bmatrix}=\begin{Bmatrix} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \\ w_1 \\ \theta_{y1} \\ w_2 \\ \theta_{y2} \end{Bmatrix} \tag{2.37}$$

The displacement in a point of the shaft is given by [62]:

$$\begin{Bmatrix} v \\ w \\ \theta_y \\ \theta_z \end{Bmatrix} = [N] \begin{Bmatrix} q_v \\ q_w \end{Bmatrix} \tag{2.38}$$

Where "N(y)" is the shape function:

$$[N] = \begin{bmatrix} [N_t] \\ [N_r] \end{bmatrix} \tag{2.39}$$

With [63]:

$$\begin{aligned} [N_t] &= \begin{bmatrix} N_{t1} & 0 & 0 & N_{t2} & N_{t3} & 0 & 0 & N_{t4} \\ 0 & N_{t1} & -N_{t2} & 0 & 0 & N_{t3} & -N_{t4} & 0 \end{bmatrix} \\ [N_r] &= \begin{bmatrix} 0 & -N_{r1} & N_{r2} & 0 & 0 & -N_{r3} & N_{r4} & 0 \\ N_{r1} & 0 & 0 & N_{r2} & N_{r3} & 0 & 0 & N_{r4} \end{bmatrix} \end{aligned} \tag{2.40}$$

The components of the shape function "N(y)" are as follows [63]:

$$\begin{aligned} N_{t1} &= \frac{1}{1+\Gamma_{comp}} \left(1 + \Gamma_{comp} - \Gamma_{comp}\, \xi - 3\xi^2 + 2\xi^3 \right) \\ N_{t2} &= \frac{L}{1+\Gamma_{comp}} \left(\frac{\left(2+\Gamma_{comp}\right)\xi}{2} - \frac{\left(4+\Gamma_{comp}\right)\xi^2}{2} + \xi^3 \right) \\ N_{t3} &= \frac{1}{1+\Gamma_{comp}} \left(\Gamma_{comp}\, \xi + 3\xi^2 - 2\xi^3 \right) \\ N_{t4} &= \frac{L}{1+\Gamma_{comp}} \left(-\frac{\Gamma_{comp}\, \xi}{2} + \frac{\left(\Gamma_{comp}-2\right)\xi^2}{2} + \xi^3 \right) \end{aligned} \tag{2.41}$$

$$\begin{aligned} N_{r1} &= \frac{6}{L\left(1+\Gamma_{comp}\right)} \left(\xi^2 - \xi \right) \\ N_{r2} &= \frac{1}{1+\Gamma_{comp}} \left(1 - 4\xi + 3\xi^2 + \Gamma_{comp} \left(1-\xi\right) \right) \\ N_{r3} &= \frac{6}{L\left(1+\Gamma_{comp}\right)} \left(-\xi^2 + \xi \right) \\ N_{r4} &= \frac{1}{1+\Gamma_{comp}} \left(3\xi^2 - 2\xi + \Gamma_{comp}\, \xi \right) \end{aligned} \tag{2.42}$$

With [63]:

$$\Gamma_{comp} = \frac{12 A_{11}}{k_s\left(A_{55} + A_{66}\right)L^2} \quad , \quad \xi = \frac{x}{L}$$

And: "Γ_{comp}" is the shear deformation parameter for composite shaft.

Moving on now to the equation of motion of the element of the shaft, by applying the Lagrange's equation on the energy expressions of the shaft, the new equation will be as follows [63]:

$$\left(\left[M_{Tc}\right]+\left[M_{Rc}\right]\right)\{\ddot{q}\}+\Omega\left[G_{Shaft}\right]\{\dot{q}\}+\left(\left[K_{Bc}\right]+\left[K_{Shear\ c}\right]+\left[K_c\right]\right)\{q\}=\{0\} \qquad (2.43)$$

Where:

- $\left[M_{Tc}\right]=\int_0^L I_m\left[N_t\right]^T\left[N_t\right]dx$ [63] is the translational mass matrix of uniform composite shaft.
- $\left[M_{Rc}\right]=\int_0^L I_d\left[N_r\right]^T\left[N_r\right]dx$ [63] is the rotational mass matrix of uniform composite shaft.
- $\left[G_{Shaft\ c}\right]=\int_0^L I_p\left[N_r\right]^T\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\left[N_r\right]dx$ [63] is the gyroscopic matrix of uniform composite shaft.
- $\left[K_{Bc}\right]=\int_0^L A_{11}\left[N_r'\right]^T\left[N_r'\right]dx$ [63]is the bending stiffness matrix of uniform composite shaft.
- $\left[K_{Shear\ c}\right]=\int_0^L \frac{L^4}{144}\Gamma_{comp}^{\ 2}\, k_s\left(A_{55}+A_{66}\right)\left[N_t'''\right]^T\left[N_t'''\right]dx$ [63] is the shear stiffness matrix of uniform composite shaft.
- $\left[K_c\right]=\int_0^L \frac{L^2}{24}\Gamma_{comp}\, k_s\, A_{16}\left[\left[N_t'''\right]^T\left[N_r'\right]+\left[N_r'''\right]^T\left[N_t'\right]\right]dx$ [63] is the geometric stiffness matrix due to axial load.

II.8 Whirl speeds analysis

The equation of motion of the system (equation 2.32) which is presented previously, can be written in the homogeneous form:

$$[M]\{\ddot{q}\}+[D+\Omega G]\{\dot{q}\}+[K]\{q\}=\{0\} \tag{2.44}$$

The new equation also can be written in the following form [63]:

$$\left[M^*\right]\{\dot{\tilde{x}}\}+\left[K^*\right]\{\tilde{x}\}=\{0\} \tag{2.45}$$

Where:

- [M*] is a positive definite and real symmetric matrix
- [K*] is an arbitrary real matrix

And they are defined by the following matrices [63]:

$$\left[M^*\right]=\begin{bmatrix}[M] & [0]\\ [0] & [I]\end{bmatrix},\ \left[K^*\right]=\begin{bmatrix}[G]+[D] & [K]\\ -[I] & [0]\end{bmatrix} \tag{2.46}$$

With: $\{\tilde{x}\}=\begin{bmatrix}\{\dot{q}\}\\ \{q\}\end{bmatrix}$ and $[I]$ is the identity matrix.

The solution of the equation (2.45) can be assumed in the form [63]:

$$\{\tilde{x}\}=\{\tilde{X}\}e^{\lambda t}=\begin{bmatrix}\{\tilde{X}_1\}e^{\lambda_1 t}\\ \{\tilde{X}_2\}e^{\lambda_2 t}\\ \vdots\\ \{\tilde{X}_n\}e^{\lambda_n t}\end{bmatrix}_{2n\times 1} \tag{2.47}$$

The eigenvalues "λ" and the eigenvectors $\{\tilde{X}\}$ are given by a new arbitrary real matrix that is obtained by substituting the equation (2.47) into equation (2.45) which gives the following form [63]:

$$\left(-[M^*]^{-1}[K^*]\right)\{\tilde{X}\}=\lambda\{\tilde{X}\} \tag{2.48}$$

So:

$$\begin{bmatrix} -[M]^{-1}\left[[D]+[G]\right] & -[M]^{-1}[K] \\ [I] & [0] \end{bmatrix}_{2n\times 2n} \begin{bmatrix} \{\tilde{X}_1\}e^{\lambda_1 t} \\ \{\tilde{X}_2\}e^{\lambda_2 t} \\ \vdots \\ \{\tilde{X}_n\}e^{\lambda_n t} \end{bmatrix}_{2n\times 1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{2n\times 1} \tag{2.49}$$

Where: $\left(-[M^*]^{-1}[K^*]\right)$ is the new arbitrary matrix.

II.9 Conclusion

In this chapter, the energy expressions of each element of the rotor are determined in order to use them on the Lagrange’s equation to obtain the general equation of motion of the rotor-bearing system, this last is based on the Timoshenko beam theory. This work is for the only purpose which is abbreviated in obtaining the elementary matrices. The modelling and analysis of rotor-bearing foundation system based on the finite element method were investigated in order to determine mass, stiffness and damping matrices of different elements to formulate accurate rotor system models.

CHAPTER III
Dynamic Analysis of Composite Rotor

III.1 Introduction

The content of the present work is the modelling and analysis of rotor system, it basically deals with theory related to dynamic analysis of composite rotor, in view of this, a theoretical model based on finite element theory will be develop, in order to write a computation program in MATLAB.

By using MATLAB program, that can handle complex matrices, the matrix procedure is demonstrated by simulation analysis, furthermore, the composite rotor system is also considered for this analysis.

Any continuous structure mathematically has an infinity of natural frequencies and mode shapes, so, to this point, some properties of the composite rotor system elements are presented in this chapter in order to create a simulation analysis for the purpose of finding the whirl natural frequency, vibration amplitudes, Campbell diagram, critical speeds and unbalance responses.

III.2 Ply plane

In this analysis, each layer of fiber can be considered as a unidirectional ply, and it is of course assumed that all plies are perfectly bounded.

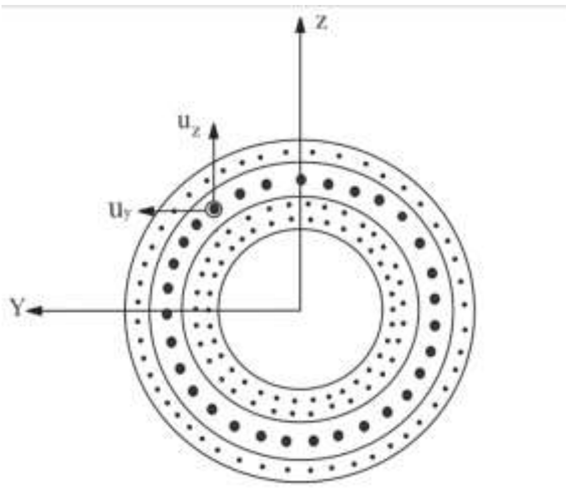

Figure III. 1: section plan [27].

III.3 Description and discretization of the studied model

1) Description of the model

In the study presented here, the composite rotor has a uniform circular shaft section with discrete isotropic rigid disk mounted at the center, it is supported by bearings that are modelled as springs and viscous dampers. The model studied is a linear system, and this system is supposed to be flexible subjected to an excitation force of an unbalance type.

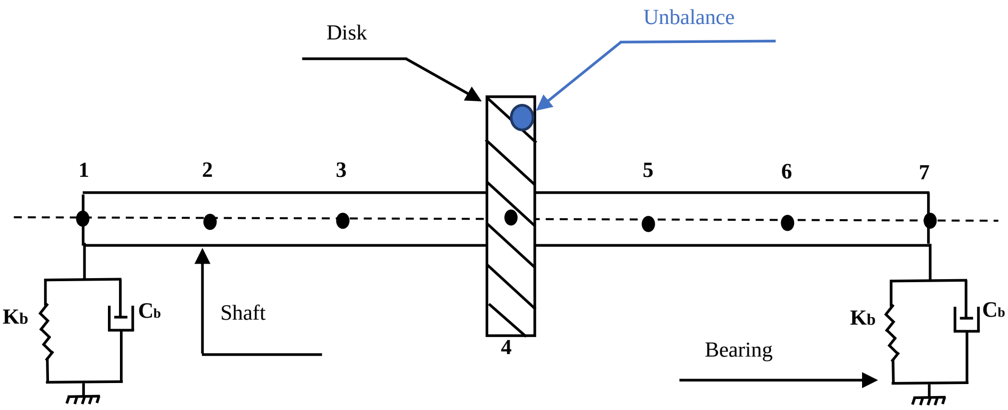

Figure III. 2: finite element model of the studied rotor.

2) Discretization of the model

The finite element method involves the discretization of a continuous structure [25], therefore, in this study, the calculation will be obtained from the discretization of the composite shaft in six elements, so, its results will be in seven nodal points.

Each element has two nodes, each node has four degrees of freedom (two translational and two rotational along the "Y" and the "Z" axis), the bearings in this system are placed on the first node (first bearing) and on the last node (second bearing), they are modelled by springs and viscous damping in the "Y" and "Z" directions, and the disk is placed on the middle of this rotor system (on the fourth node).

Thus, the size of the global matrix will be obtained from the multiplication of the number of nodes by degrees of freedom : "7*4=28" (28*28), and after the boundary conditions will be applied (eliminate the two degrees of freedom of the bearings), the global matrix takes the size of (24*24).

The table below shows the numerical data concerning the discretization of the structure:

	Elements	Nodes	DOFs
Rotor	6	7	28
Application of the boundary conditions	6	7	24

Table III. 1: Discretization.

III.4 Properties of the elements of the rotor

It is important to know the properties of the rotor's elements to study the behavior of a structure, so, the geometric properties of a composite shaft and a rigid disk as well as the bearings are represented on the tables below:

- Composite shaft:

Properties	Values
Total length	L=0.72m
Inner diameter	ID=0.028m
Outer diameter	OD=0.048m
Lay-up from inside	(90°,45°, -45°, 0_6°, 90°)
Shear correction factor	k_s=0.56

Table III. 2: Properties of the shaft [63].

- Rigid disk:

Properties	Values
Mass	m=2.4364 kg
Diametral mass moment of inertia	I_d=0.1901 kg.m²
Polar mass moment of inertia	I_p=0.3778 kg.m²
The mass eccentricity of the disk	e=5*10^-5

Table III. 3: Properties of the disk [63].

- Bearings:

Properties	Values
The stiffness	K_{yy}=K_{zz}=1.75*10^7 N/m
The damping	C_{yy}=C_{zz}=5*10^2 N.s/m

Table III. 4: Properties of the bearings [63].

III.5 Boundary conditions

The discussion of the boundary conditions is necessary and it must be introduced, they are abbreviated in cancelling the degrees of freedom. The two boundary conditions of this system are:

- Along the "X" axis, the two degrees of freedom are cancelled, so, it'll be last the four degrees of freedom along the "Y" and the "Z" axis (two translational and two rotational DOFs)
- On the bearings, only two degrees of freedom are considered (two rotational DOFs along the "Y" and the "Z" axis)

The boundary conditions are considered only for the purpose of solving the system presented in this study.

III.6 Numerical models

The finite element model of a composite shaft is calculated by using a numerical example contains two composite materials "boron/epoxy" and "graphite/epoxy", which are considered for the vibration analysis [63]. The results of those two composite materials can be compared to each other, their calculations is obtained from MATLAB program developed to perform the vibration analysis of composite shaft [63]. In this study, each element will be modelled and represented by eight degrees of freedom with taking the gyroscopic effect into account. The table below shows some mechanical properties of the two composite materials:

Properties		**Materials**	
		Boron/epoxy	Graphite/epoxy
Young modulus	E11	211 GPa	139 GPa
	E22	24 GPa	11 GPa
Shear modulus	G12=G13	6.9 GPa	6.05 GPa
	G23	6.9 GPa	3.78 GPa
Poisson coefficient (ν)		0.36	0.313
The density (ρ)		1967 Kg/m^3	1578 Kg/m^3

Table III. 5: Properties of composite materials [63].

III.7 Composite rotor simulation model

The calculation program developed in this work, is a model written in MATLAB/ SIMULINK, this program consists in solving the dynamic equations of a composite rotor system, and, the SIMULINK model is developed in order to solve the equations of motion [64].

The simulation model of composite rotor has made to determine the natural frequencies of a rotational composite rotor with different boundary conditions and different physical and geometric parameters [23].

1) Resolution system

The different programming steps are as follows:

- Reading the data of all the necessary elementary physical and geometric parameter of the system (shaft, disk and bearings).
- Reading all the values of the integrals.
- Formation of elementary matrices (chapter 2).
- Formations of global matrices [M], [K], [G] and [C] with taking the boundary conditions into account.
- Formations of reduced matrices [Mr], [Kr], [Gr].
- The programme gives the results of the eigenvalues (ω).

2) Organizational chart

There are many methods for solving systems of equations, each one has its interest according to the phenomenon to which it is treated. After the elementary matrices have been expressed (on chapter 2), assembly global matrices are carried out to build the system of equation [27], The following organizational chart shows the steps to resolve a dynamic problem:

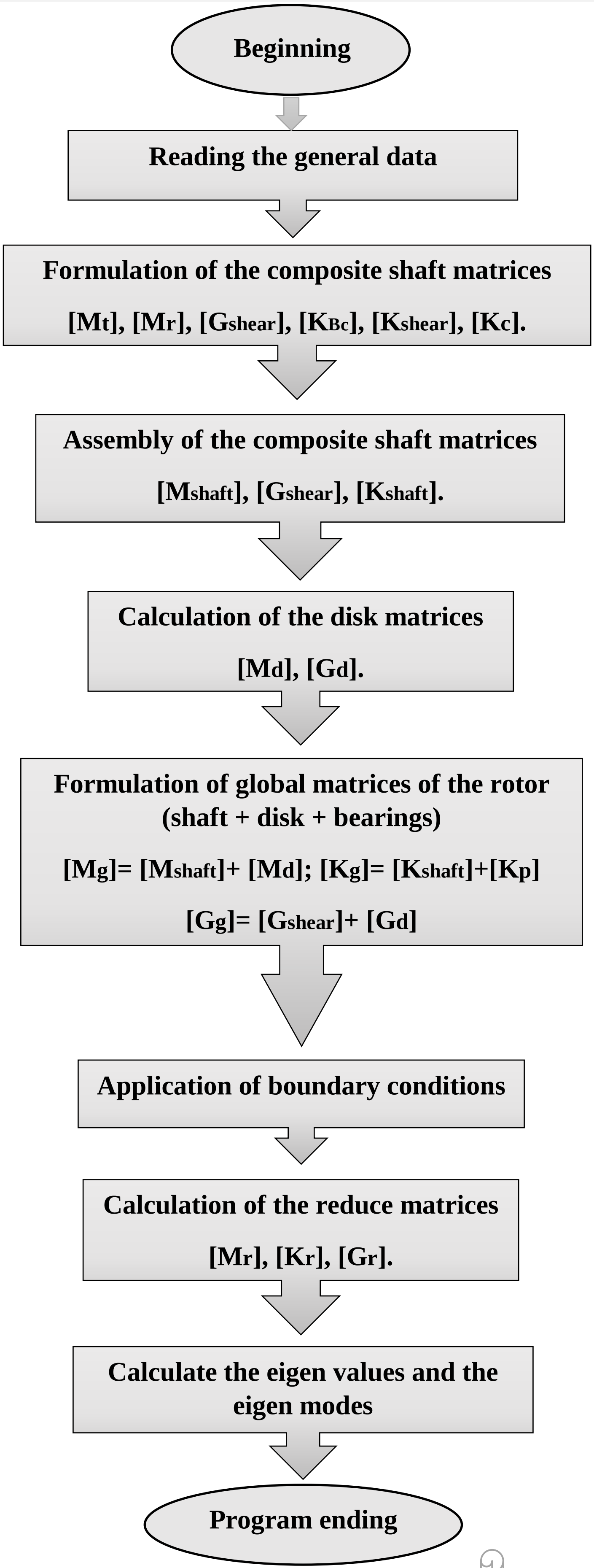

Figure III. 3: Organizational chart.

III.8 Results and discussions

The MATLAB/SIMULINK program is written to perform the vibration analysis of the uniform composite shaft in order to validate the finite element model.

1) Comparison between two composite materials

a. Campbell Diagram

The Campbell diagram is one of the most important tools for understanding the dynamic of the rotating machines, it is used to evaluate the critical speed at different operating speed [65]. The **"Figure III.4"** represents the critical speeds as function of rotating speed for both of composite materials.

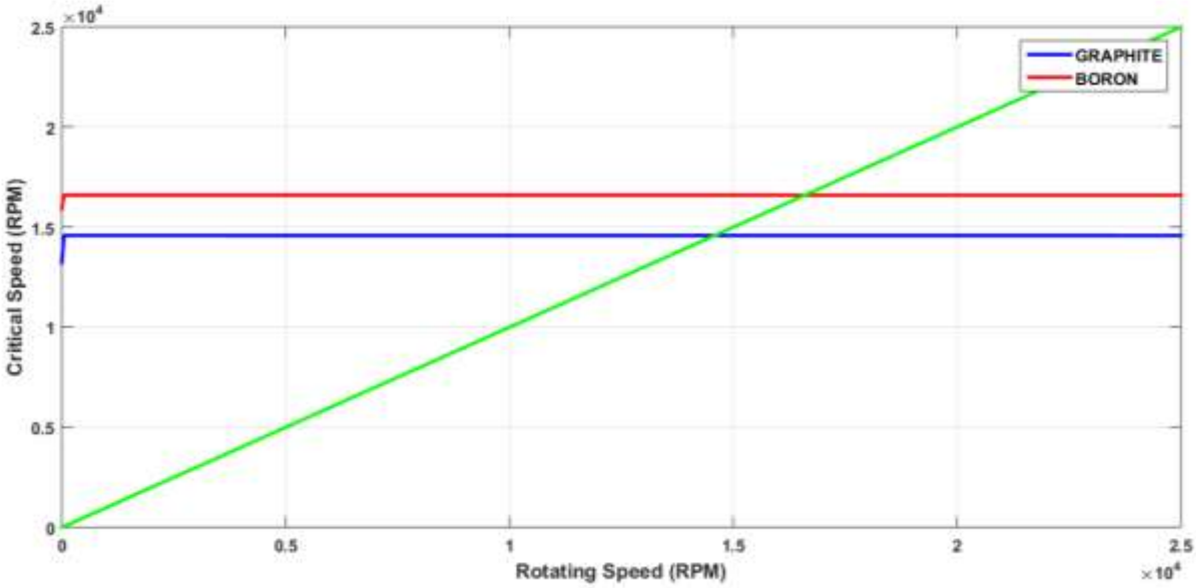

Figure III. 4: Campbell diagram of critical speeds as function of rotating speeds.

The critical speeds obtained from the present example are shown in the table below:

Materials	Graphite/Epoxy	Boron/Epoxy
Critical speeds (Krpm)	14.5	16.5

Table III. 6: Critical speeds of the two different composite materials

In this example, Campbell diagram maps two critical speeds which are caused by the influence of the shear deformation effect, the composite shaft made of "graphite/epoxy" has the first critical speed "N_{CG}", and the composite shaft made of "boron/epoxy" has the second critical speed "N_{CB}".

b. Mode shape

As it is known that rotating machinery has to rotate to do useful work, the plot below shows what happens to the first mode of this rotor system once it is spinning [66].

"Figure III.5" illustrates the vibration mode shape on different parts of the structure.

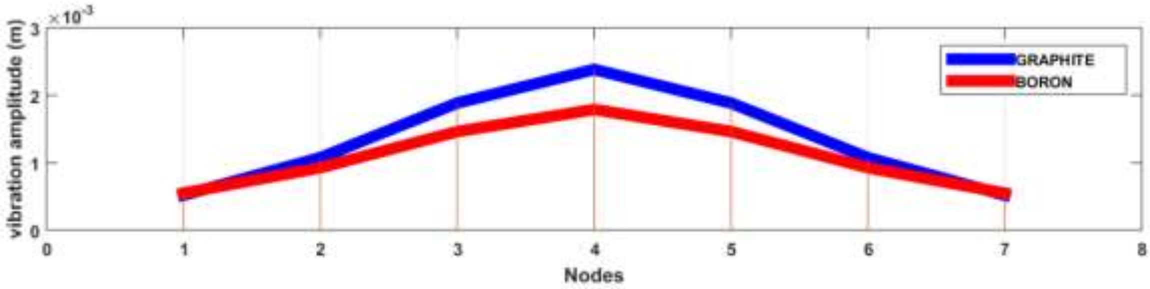

Figure III. 5: The first mode shape.

Note that the vibration amplitudes of both composite materials are obtained through only the first mode, and it is noticed that those amplitudes are higher at the level of the disk. The second and the third mode are not considered on this rotor system, which means that the using of composite materials on the rotor dynamics is important.

c. Vibration amplitudes of disk

The vibration amplitudes for both of composite materials as function of rotational speed are illustrated in the following figure **"Figure III.6".**

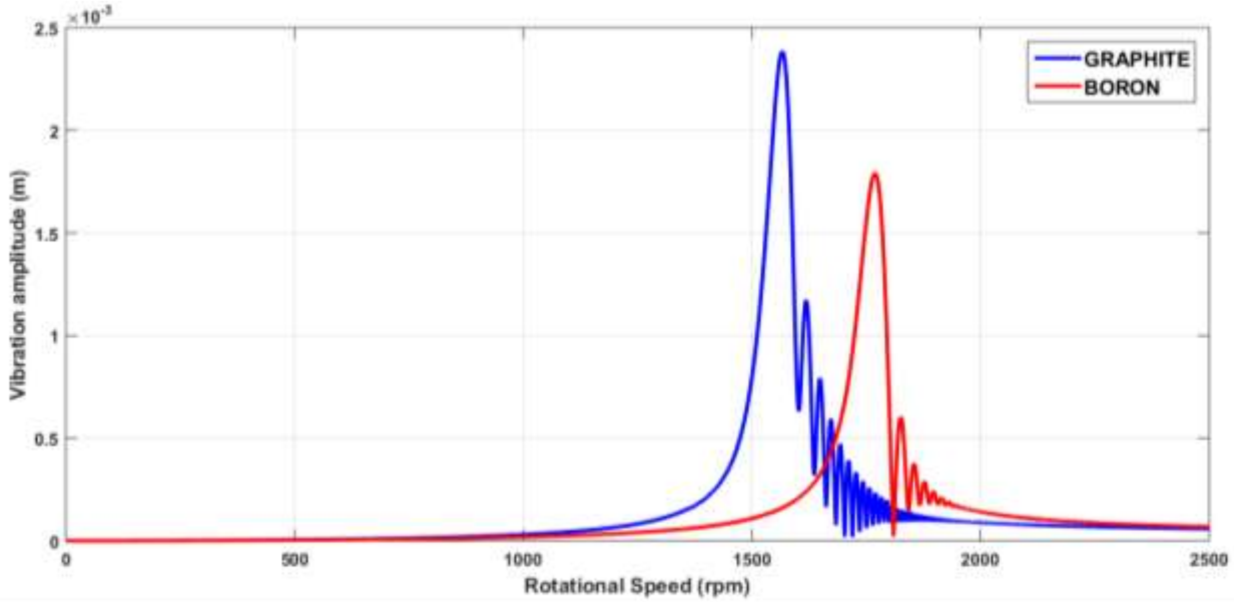

Figure III. 6: Vibration amplitudes of a rigid disk for the two composite materials.

First of all, this plot has two zones, the rigid zone on the operating range, and it is before the peaks, and the flexible zone which is after the peaks. Note that the "graphite/epoxy" material has the first vibration amplitude peak "A_{CG}" which is bigger than the second peak "A_{CB}" of the "boron/epoxy".

$$A_{CG} = 2.45\times10^{-3}m,\quad A_{CB} = 1.75\times10^{-3}m$$

$$A_{CG} > A_{CB}$$

The difference between these amplitude values is obtained with a reducing ratio which is determined as follows:

$$\zeta_B = \frac{x}{A_{CG}}\times100 \quad , \quad x = A_{CG} - A_{CB} = (2.45-1.75)\times10^{-3} = 0.7\times10^{-3}m$$

$$\Rightarrow \zeta_B = \frac{0.7\times10^{-3}}{2.45\times10^{-3}}\times100 = 28.57\,\%$$

d. Vibration amplitude of the bearings

For the bearings, the evolution of the vibration amplitudes as function of the rotational speed is presented in the **"Figure III.7".**

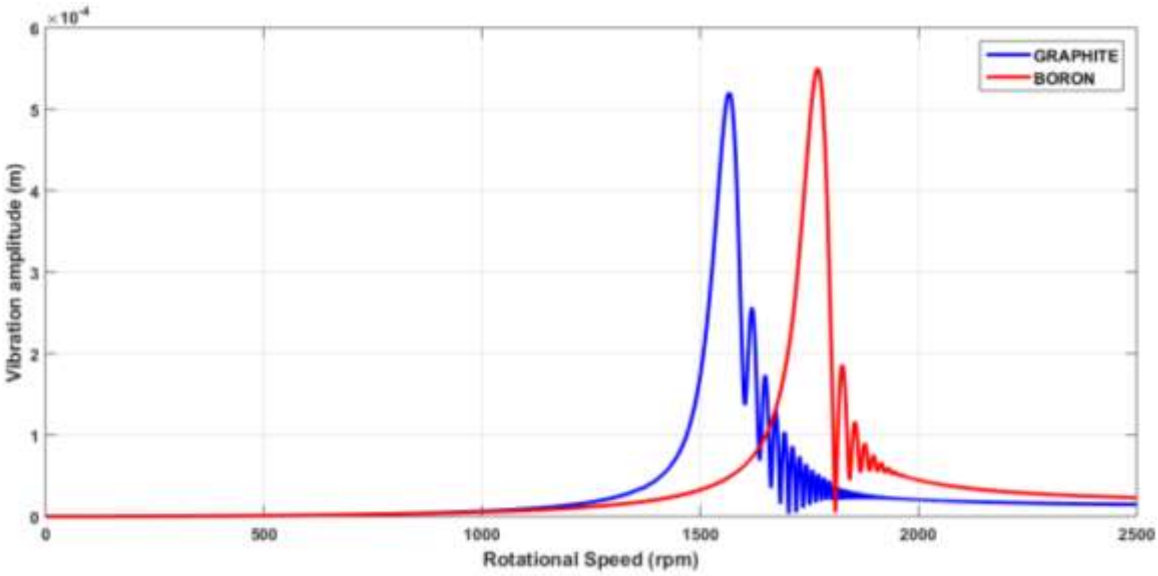

Figure III. 7: Vibration amplitudes of the bearings for the two composite materials.

It is noticed that "graphite/epoxy" material has the first vibration amplitude peak of "$5.2*10^{-4}$ m" which is lower than the "boron/epoxy" one that has a peak of "$5.5*10^{-4}$ m". this difference is around to 5.45% ((5.5-5.2)/ 5.5). on the other hand, it is noticed that the damping ratio of the "graphite/epoxy" is much lower than the "boron/epoxy" one, so that can be explained the stiffness of Boron is very higher than Graphit stiffness

$$"\zeta_B > \zeta_G" \quad \Leftrightarrow \frac{D_B}{2\sqrt{K_B m_B}} > \frac{D_G}{2\sqrt{K_G m_G}}$$

And since: $K_B >> K_G$, so: "$D_B > D_G$"

Where: "D" is the damping.

e. Transmitted force

The **"Figure III.8"** represents the variation of the amplitude of the transmitted force as function of the rotational speed.

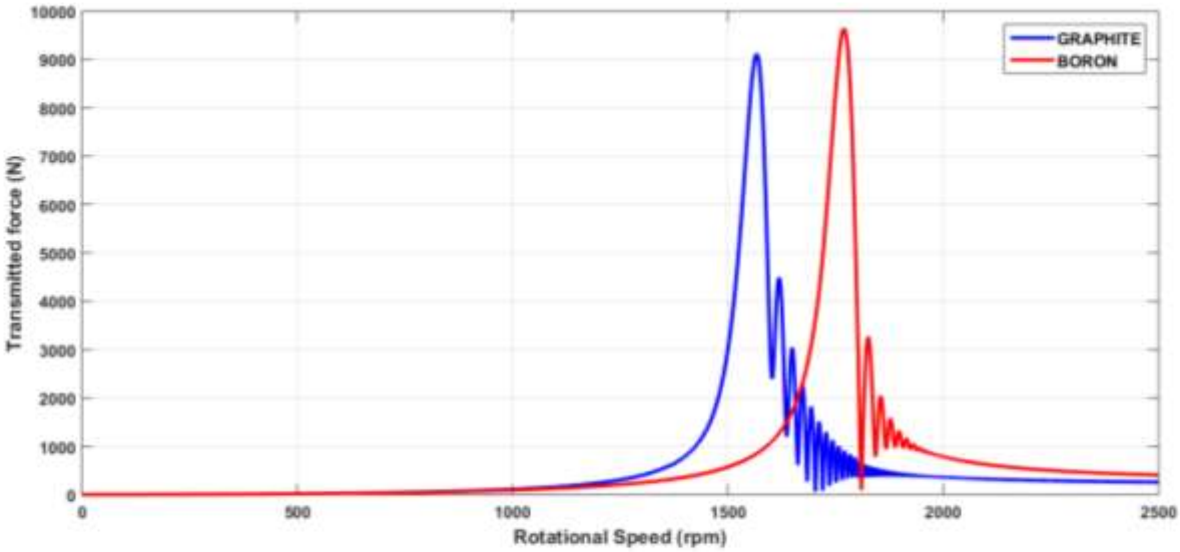

Figure III. 8: Transmitted force due to rotation unbalance.

This plot shows that there are two amplitude's peaks of the transmitted force at speeds close to a critical speed. The first peak is for the "graphite/epoxy" and the second one is for the "boron/epoxy". Note that those peaks are close to each other so the reduction ratio between them is literally negligible.

Those transmitted force amplitudes are obtained by the hydrostatic forces which are given by the following equations:

$$\left.\begin{array}{l} F_{Tx} = K_x\, x + D_x\, \dot{x} \\ F_{Ty} = K_y\, y + D_y\, \dot{y} \end{array}\right\} \Rightarrow \left\{ F_T = \sqrt{{F_{Tx}}^2 + {F_{Ty}}^2} \right\}$$

In order to have a significant reduction in the transmitted force, it has to be a proper design of a bearing support system [28].

After the interpretation of the previous plots, the results illustrate that the shafts made of "boron/epoxy" have higher mechanical properties (stiffness, damping and damping ration) than the shafts made of "graphite/epoxy", which can explain that the "boron/epoxy" gives better results.

So, in the next interpretations and analysis, only "boron/epoxy" material is considered.

2) The unbalance effect on dynamic behavior

The **"Figures III.9, 10"** below represent orbits of vibration amplitudes as function of different eccentricity values (e=30µm, e=50µm, e=70µm), in the middle shaft and in the bearing journal respectively at the speed of shaft made by "boron/epoxy".

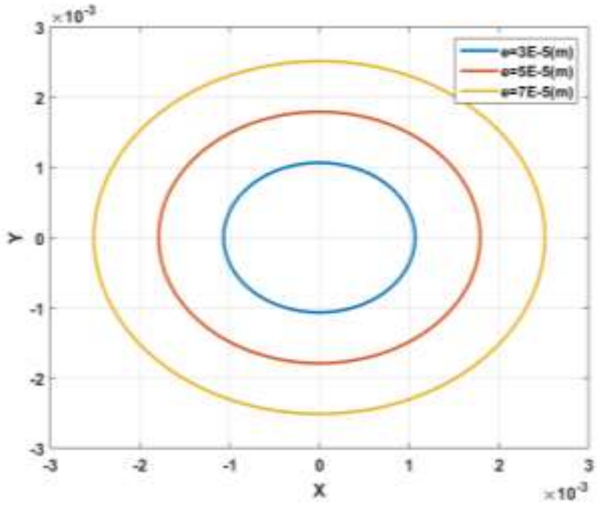

Figure III. 9: The effect of the unbalance eccentricity on vibration amplitude orbits in middle shaft at "NCB".

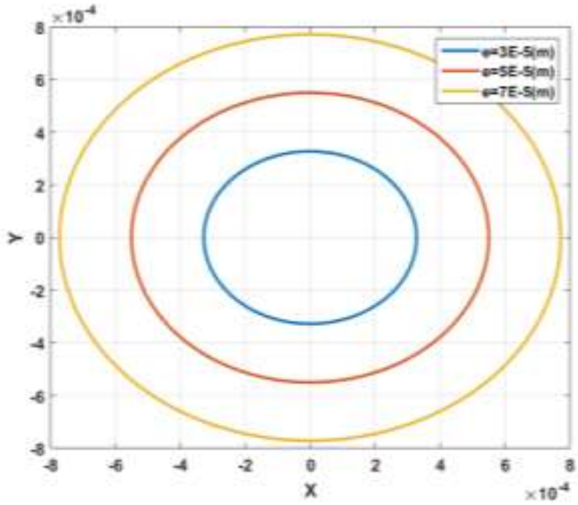

Figure III. 10: The effect of the unbalance eccentricity on vibration amplitude orbits in bearing journal at "NCB

The **"Figure III.11"** represent orbits of bearing transmitted forces as function of different eccentricity values (e=30µm, e=50µm, e=70µm), at the speed of shaft made by "boron/epoxy".

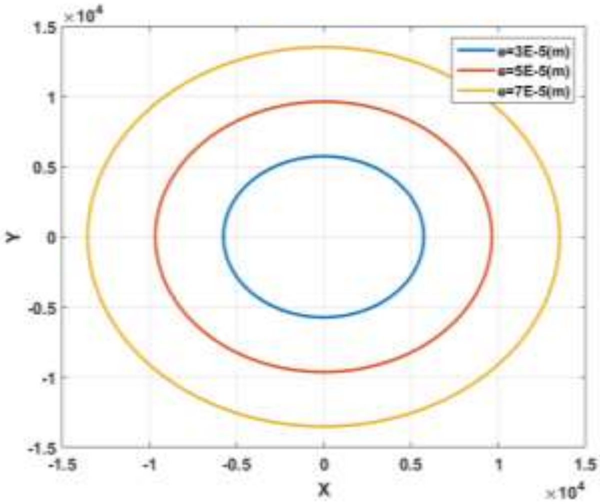

Figure III. 11: The effect of the unbalance eccentricity on bearing transmitted force orbits at "NCB".

The results show that the vibration amplitudes in middle shaft and in bearing journal, increase with unbalance eccentricity, as well as the bearing transmitted forces, and this is due to the increase of unbalance forces.

III.9 Conclusion

Rotor dynamics analysis including determination of critical speeds, vibration amplitudes and steady-state response, is conducted through numerical simulation works. In this chapter, the results obtained concerning the Campbell diagram, the variation of the vibration amplitudes, and the influence of the unbalance eccentricity on vibration amplitudes and bearing transmitted forces, are presented and interpreted.

GENERAL CONCLUSION

Vibration analysis of a rotating composite shaft using the finite element model that is based on the Timoshenko beam theory, is presented in this work with the help of the "LAGRANGE" formulation to solve the equation of motions.

The utilization of the finite element model in the aera of rotor dynamics has yielded highly great results, and it has been successful in solving problems with complicated geometry and without the need to accept many simplifying assumptions, and the MATLAB programming helps to have fast calculations with high efficiency. There are several materials that shafts can be made of, among them, we chose in this study the graphite/epoxy and the boron/epoxy, on the purpose of making a comparison between them, so, the results of the numerical model obtained show that:

- ✓ Rotors made of composite material don't need to reach the second and the third mode shape to obtain the vibration amplitudes, the first mode shape is enough.
- ✓ Speaking about "boron/epoxy" material, it has higher mechanical properties (stiffness, damping and damping ratio) than the "graphite/epoxy", this is what makes it the interest of this study.
- ✓ Transmitted force to bearings due to rotation unbalance, has influenced by the unbalance eccentricity, and they have a direct correlation between them, as the transmitted force increases with the increase of the eccentricity.

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ABSTRACT

Rotor dynamic is a branch of applied mechanics that plays a major role in keeping the vibrational energy as small as possible. It covers several topics, two of them are the base of this study: modelling and analysis. The use of composite materials in rotor dynamic analysis gives an important calculation to design efficient rotating composite shaft. The energy expressions of a composite rotor system are obtained by the use of rotating coordinate system. Lagrange's equation obtains the equation of motions, and the finite element method solves those equations to obtain the matrices of the rotor system with the use of the MATLAB program that can handle complex and large matrices. Those mathematical models are used in order to explain the dynamic behavior of composite rotor. This model is used to investigate the unbalance response of a rotating composite shaft supported by flexible journal bearings in transient regime. The numerical results show that composite shaft made of Boron/Epoxy have better dynamic characteristics than Graphite/Epoxy.

Key words: Rotor dynamics, Composite materials, Finite element model, Vibration analysis, Energy expressions, Natural frequencies.

Résumé

La dynamique du rotor est une branche de la mécanique appliquée qui joue un majeur rôle dans maintien de l'énergie vibratoire aussi petite que possible. Il couvre plusieurs sujets, dont deux sont la base de cette étude : la modélisation et l'analyse. L'utilisation des matériaux composites dans l'analyse dynamic de rotor donne un calcul important pour concevoir un efficace rotatif arbre en composite. Les expressions d'énergie d'un système du rotor en composite sont obtenues par l'utilisation d'un system de coordonnées de rotation. L'équation de Lagrange obtient l'équation des mouvements, et la méthode des éléments finis résout ces équations pour obtenir les matrices du système de rotor avec l'utilisation de programme MATLAB qui peut gérer des matrices complexes et volumineuses. Ces modelés mathématiques sont utilisés pour expliquer le comportement dynamique du rotor en composite. Ce modèle est utilisé pour étudier la réponse au déséquilibre d'un arbre composite rotatif supporté par des paliers lisses flexibles en régime transitoire.Les résultats numériques montrent que l'arbre composite en Bore / Epoxy a de meilleures caractéristiques dynamiques que le Graphite / Epoxy.

Mots clé : Dynamique des rotors, Matériaux composites, Model des élément finis, Analyse vibratoire, Expression d'énergie, Fréquences naturelles.

ملخص

الدوار الديناميكي هو فرع من الميكانيك التطبيقية، يلعب دورا رئيسيا في الحفاظ على الطاقة الاهتزازية صغيرة قدر الإمكان. يشمل الدوار الديناميكي عدة مواضيع، اثنان منها هما أساس هذه الدراسة: النمذجة والتحليل. استخدام المواد المركبة في تحليل الدوار الميكانيكي يعطي عملية حسابية مهمة لتصميم عمود دوار مركب فعال. يتم الحصول على عبارات الطاقة لنظام الدوار المركب عن طريق استخدام نظام احداثيات الدوران. تتحصل معادلة Lagrange على معادلة الحركات، وتقوم طريقة العناصر المحدودة بحل تلك المعادلات للحصول على مصفوفات النظام الدوار باستخدام برنامج MATLABالذي يمكنه التعامل مع المصفوفات المعقدة والكبيرة. يتم استخدام هذه النماذج الرياضية لشرح السلوك الديناميكي للعضو الدوار المركب. يستخدم هذا النموذج للتحقيق في الاستجابة غير المتوازنة لعمود مركب دوار مدعوم بمحامل مجلة مرنة في نظام عابر. يستخدم هذا النموذج لفحص الترددات الطبيعية واستقرار النظام. بعد التحليل الاهتزازي. تظهر النتائج العددية أن العمود المركب Boron/Epoxy له خصائص ديناميكية أفضل Graphite/Epoxy.

الكلمات المفتاحية: الدوار الديناميكي، المواد المركبة، نموذج العناصر المحدودة، التحليل الاهتزازي، عبارات الطاقة، ترددات الطبيعية.