

Annexe d'application

```
> restart;
```

Entrée des données

```
> E:=72;G:=28;
```

$$E := 72$$

$$G := 28$$

```
> rho:=2700;
```

$$\rho := 2700$$

```
> nu:=0.33;
```

$$\nu := 0.33$$

```
> s11:=1/E;s12:=-nu/E;s44:=1/G;
```

$$s11 := \frac{1}{E}$$

$$s12 := -\frac{\nu}{E}$$

$$s44 := \frac{1}{G}$$

Matrice de souplesse [S]

```
> s:=array([[s11,s12,s12,0,0,0],[s12,s11,s12,0,0,0],[s12,s12,s11,0,0,0],[0,0,0,s44,0,0],[0,0,0,0,s44,0],[0,0,0,0,0,s44]]);
```

$$s := \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix}$$

Matrice de rigidité [C]

> **c:=evalm(s**(-1));**

$$c := \begin{bmatrix} \frac{E(v-1)}{2v^2+v-1} & -\frac{Ev}{2v^2+v-1} & -\frac{Ev}{2v^2+v-1} & 0 & 0 & 0 \\ -\frac{Ev}{2v^2+v-1} & \frac{E(v-1)}{2v^2+v-1} & -\frac{Ev}{2v^2+v-1} & 0 & 0 & 0 \\ -\frac{Ev}{2v^2+v-1} & -\frac{Ev}{2v^2+v-1} & \frac{E(v-1)}{2v^2+v-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix}$$

Les éléments C_{ij} de [C]

> **c11:=c[1,1];c12:=c[1,2];c44:=c[4,4];c22:=c11;c33:=c11;c13:=c12**
;c23:=c12;c55:=c44;c66:=c44;c14:=0;c15:=0;c16:=0;c24:=0;c25:=0
;c26:=0;c34:=0;c35:=0;c36:=0;c45:=0;c46:=0;c56:=0;

$$c11 := \frac{E(v-1)}{2v^2+v-1}$$

$$c12 := -\frac{Ev}{2v^2+v-1}$$

$$c44 := G$$

$$c22 := \frac{E(v-1)}{2v^2+v-1}$$

$$c33 := \frac{E(v-1)}{2v^2+v-1}$$

$$c13 := -\frac{Ev}{2v^2+v-1}$$

$$c23 := -\frac{Ev}{2v^2+v-1}$$

$$c55 := G$$

$$c66 := G$$

$$c14 := 0$$

$$c15 := 0$$

$$c16 := 0$$

$$c24 := 0$$

$$c25 := 0$$

$$c26 := 0$$

$$c34 := 0$$

$$c35 := 0$$

$$c36 := 0$$

$$c45 := 0$$

$$c46 := 0$$

$$c56 := 0$$

Les a_{ij} :

>a1:=-c13/c33;a2:=c11-(c13**2)/c33;a3:=c12-(c13*c23)/c33;a4:=c22-(c23**2)/c33;a5:=-c23/c33;a6:=c66;a7:=1/c33;a9:=1/c44;a10:=c44/(c44*c55-(c45**2));a11:=c45/(c44*c55-(c45**2));a12:=c55/(c44*c55-(c45**2));a13:=c66-(c36**2)/c33;a14:=c16-(c13*c36)/c33;a15:=c26-(c23*c36)/c33;a16:=-c36/c33;

$$a1 := -\frac{c13}{c33}$$

$$a2 := c11 - \frac{c13^2}{c33}$$

$$a3 := c12 - \frac{c13 c23}{c33}$$

$$a4 := c22 - \frac{c23^2}{c33}$$

$$a5 := -\frac{c23}{c33}$$

$$a6 := c66$$

$$a7 := \frac{1}{c33}$$

$$a9 := \frac{1}{c44}$$

$$a10 := \frac{c44}{c44 c55 - c45^2}$$

$$a11 := \frac{c45}{c44 c55 - c45^2}$$

$$a12 := \frac{c55}{c44 c55 - c45^2}$$

$$a13 := c66 - \frac{c36^2}{c33}$$

$$a14 := c16 - \frac{c13 c36}{c33}$$

$$a15 := c26 - \frac{c23 c36}{c33}$$

$$a16 := -\frac{c36}{c33}$$

Introduction des variables en vibration libre

> **sx0:=0;**

$$sx0 := 0$$

> **b:=a;**

$$b := a$$

> **R:=0.1;h:=1;**

$$R := 0.1$$

$$h := 1$$

> **m:=1;n:=1;**

$$m := 1$$

$$n := 1$$

> **a:=m*h/R;**

$$a := 10.00000000$$

> **nd:=2;**

$$nd := 2$$

> **tau:=h/nd;**

$$\tau := \frac{1}{24}$$

> **p1:=m*Pi/a; p:=evalf(p1) ;q1:=n*Pi/b; q:=eval(q1);**

$$p1 := \frac{m \pi}{a}$$

$$p := \frac{3.141592654 m}{a}$$

$$q1 := \frac{n \pi}{b}$$

$$q := \frac{3.141592654 n}{b}$$

Sous matrice [q]

>**q11:=1;q12:=0;q13:=tau*(-p);q21:=0;q22:=1;q23:=tau*(-q);q31:=tau*(-a1*p+a16*q);q32:=tau*(-a5*q+a16*p);q33:=1;**

$$q11 := 1$$

$$q12 := 0$$

$$q13 := -\tau p$$

$$q21 := 0$$

$$q22 := 1$$

$$q23 := -\tau q$$

$$q31 := \tau(-a1 p + a16 q)$$

$$q32 := \tau(-a5 q + a16 p)$$

$$q33 := 1$$

Sous matrice [r]

>r11:=0;r12:=tau*(a10);r13:=tau*(-a11);r21:=0;r22:=tau*(-a11);r23:=tau*(a12);r31:=tau*(a7);r32:=0;r33:=0;

$$r11 := 0$$

$$r12 := \tau a10$$

$$r13 := -\tau a11$$

$$r21 := 0$$

$$r22 := -\tau a11$$

$$r23 := \tau a12$$

$$r31 := \tau a7$$

$$r32 := 0$$

$$r33 := 0$$

>alpha:=(a2*(p**2)+a13*(q**2)+2*a14*p*q);beta:=(a14*(p**2)+a15*(q**2)+(a13+a3)*p*q);Gamma:=(a13*(p**2)+a4*(q**2)+2*a15*p*q);

$$\alpha := a2 p^2 + a13 q^2 + 2 a14 p q$$

$$\beta := a14 p^2 + a15 q^2 + (a13 + a3) p q$$

$$\Gamma := a13 p^2 + a4 q^2 + 2 a15 p q$$

Sous matrice [t]

>t11:=0;t12:=0;t13:=tau*((p**2)*sx0-rho*ww);t21:=tau*(alpha-rho*ww);t22:=tau*(beta);t23:=0;t31:=t22;t32:=tau*(Gamma-rho*ww);t33:=0;

$$t11 := 0$$

$$t12 := 0$$

$$t13 := \tau(p^2 sx0 - \rho ww)$$

$$t21 := \tau(a2 p^2 + a13 q^2 + 2 a14 p q - \rho ww)$$

$$t22 := \tau(a14 p^2 + a15 q^2 + (a13 + a3) p q)$$

$$t23 := 0$$

$$t31 := \tau(a14 p^2 + a15 q^2 + (a13 + a3) p q)$$

$$t32 := \tau(a13 p^2 + a4 q^2 + 2 a15 p q - \rho w w)$$

$$t33 := 0$$

Sous matrice [d]

> **d11:=1;d12:=tau*(p);d13:=tau*(q);d21:=tau*(a1*p+a16*q);d22:=1;
d23:=0;d31:=tau*(a5*q+a16*p);d32:=0;d33:=1;**

$$d11 := 1$$

$$d12 := \tau p$$

$$d13 := \tau q$$

$$d21 := \tau(a1 p + a16 q)$$

$$d22 := 1$$

$$d23 := 0$$

$$d31 := \tau(a5 q + a16 p)$$

$$d32 := 0$$

$$d33 := 1$$

Matrice [H]

> **H:=array([[q11,q12,q13,r11,r12,r13], [q21,q22,q23,r21,r22,r23],
[q31,q32,q33,r31,r32,r33], [t11,t12,t13,d11,d12,d13], [t21,t22,t
23,d21,d22,d23], [t31,t32,t33,d31,d32,d33]]);**

$$H := \begin{bmatrix} 1, & 0, & -\tau p, & 0, & \tau a10, & -\tau a11 \\ 0, & 1, & -\tau q1, & 0, & -\tau a11, & \tau a12 \\ \tau(-a1 p + a16 q1), & \tau(-a5 q1 + a16 p), & 1, & \tau a7, & 0, & 0 \\ 0, & 0, & \tau(p^2 s x0 - \rho w w), & 1, & \tau p, & \tau q1 \\ \tau(\alpha - \rho w w), & \tau \beta, & 0, & \tau(a1 p + a16 q1), & 1, & 0 \\ \tau \beta, & \tau(\Gamma - \rho w w), & 0, & \tau(a5 q1 + a16 p), & 0, & 1 \end{bmatrix}$$

Transformation du [H] par différence finie :

> **T:=evalm(H**nd);**

Matrice [T21]

```
>T21:=array([[T[4,1],T[4,2],T[4,3]],[T[5,1],T[5,2],T[5,3]],[T[6,1],T[6,2],T[6,3]]]);
```

Déterminant de [T21]

```
>F1:=T[4,1]*(T[5,2]*T[6,3]-T[5,3]*T[6,2])-
T[4,2]*(T[5,1]*T[6,3]-T[5,3]*T[6,1])+T[4,3]*(T[5,1]*T[6,2]-
T[5,2]*T[6,1]);
```

```
>F:=factor(F1);
```

Résolution du problème propre T21=0

```
>solve({F=0},{w});
```

Fréquences naturelles :

```
>ww1:=.9173002062e-4;
```

ww1 := 0.00009173002062

```
>wb:=ww1*rho/E;
```

wb := 0.003439875774

```
>wbar:=wb**(1/2);
```

wbar := 0.05865045417